Computer Graphics

Lecture-08
Three - Dimensional Transformation

Md Imtiaz Ahmed
Lecturer,
DIIT

3D Point

- We will consider points as column vectors. Thus, a typical point with coordinates (x, y, z) is represented as:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

3D Point

- A 3D point **P** with coordinates (x, y, z) is represented as:

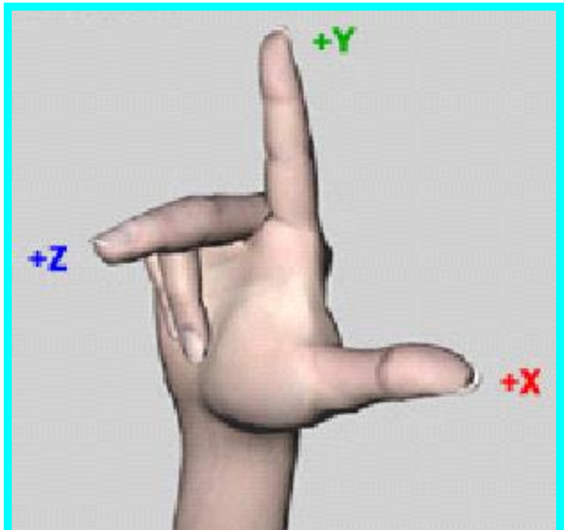
$$P = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

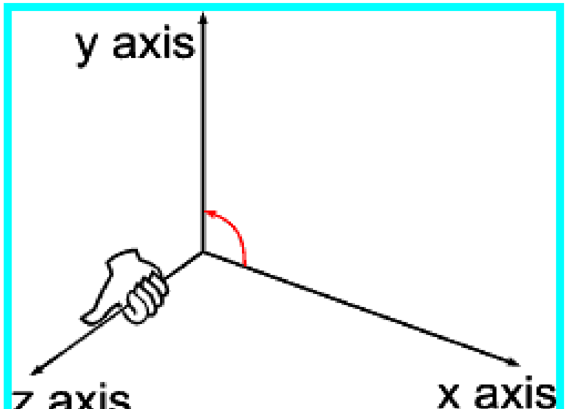
3D Transformation

- 3D transformations are represented by 4×4 matrixes:

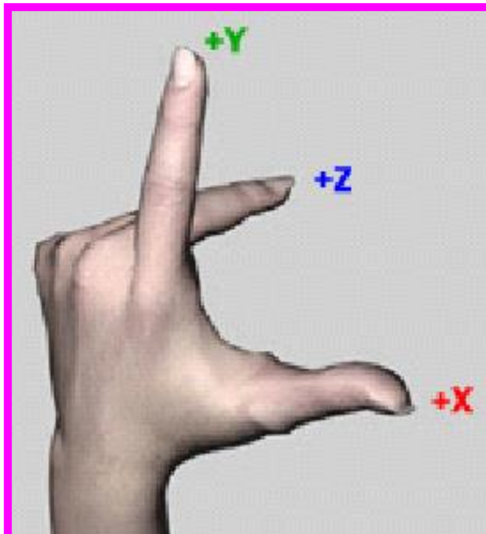
3D Coordinate Systems

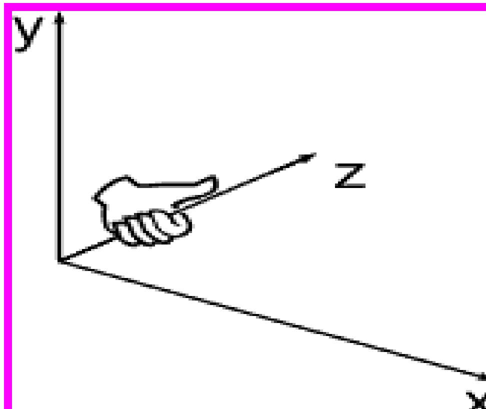
- ***Right Hand***
  coordinate system:





- ***Left Hand***
  coordinate system:





Three - Dimensional Transformation

- Manipulation, viewing and construction of three-dimensional graphic images requires the use of three dimensional geometric and coordinate transformations.
- These transformations are formed by composing the basic transformations of translation, scaling and rotation.
- Each of these transformations can be represented as a matrix transformation.
- Transformations are now represented as 4x4 matrices
- Geometric transformations
- Coordinate transformations
- Composite transformations
- Instance transformations

Three - Dimensional Transformation

- Very similar to 2D transformation

- Scaling transformation

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Geometric transformations

- Object moves
- Coordinate system remains stationary
- Basic transformations are translation, scaling and rotation.

- Translation

$$V' = V + D$$

- Scaling

$$V' = SV$$

- Rotation

$$V' = RV$$

Translation

- The amount of the translation is added to or subtracted from the x, y, and z coordinates
- In general, this is done with the equations:

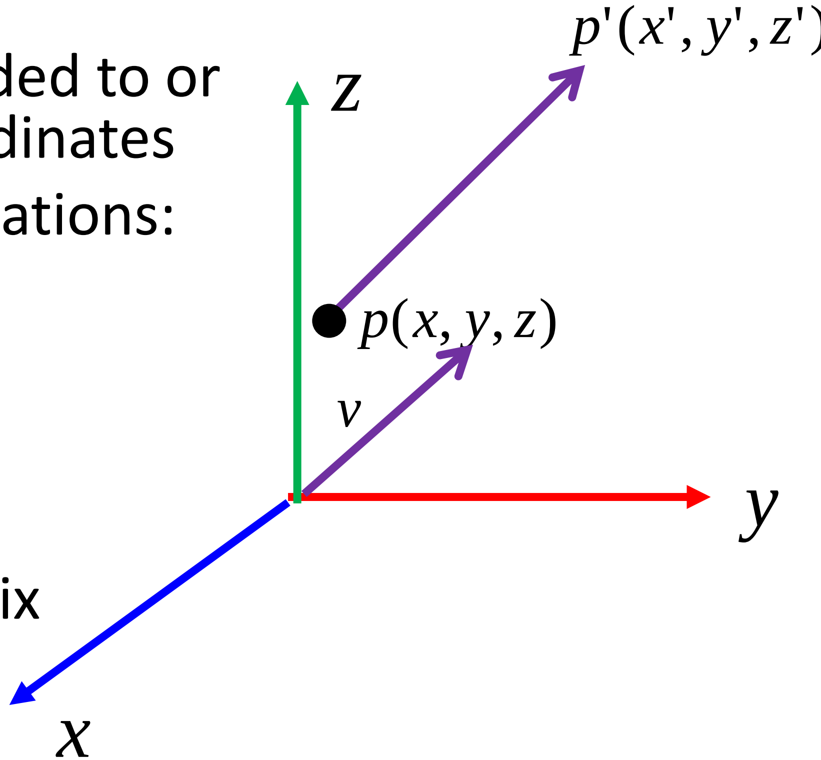
$$x' = x + T_x$$

$$y' = y + T_y$$

$$z' = z + T_z$$

- This can also be done with the matrix multiplication:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Scaling

- In general, this is done with the equations:

  $x_n = s_x * x$

  $y_n = s_y * y$

  $z_n = s_z * z$

- This can also be done with the matrix multiplication:

$$\begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scaling

- Scaling

$$V' = SV$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

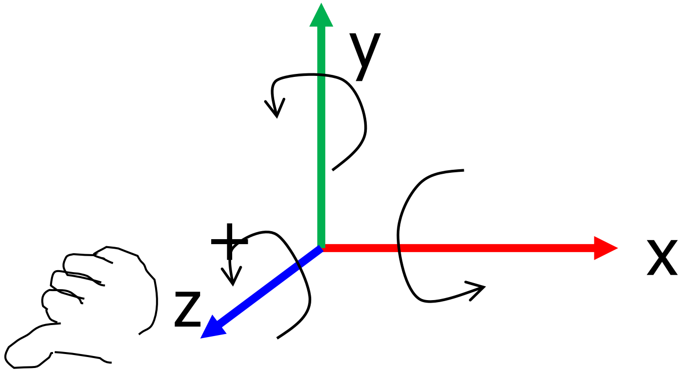
$$x' = x \cdot S_x$$

$$y' = y \cdot S_y$$

$$z' = z \cdot S_z$$

Rotation

- 3D rotation is done around a rotation **axis**
- Fundamental rotations – rotate about x, y, or z axes
- Counter-clockwise rotation is referred to as positive rotation .



Rotation

- The matrix form for rotation

– x axis

$$\begin{bmatrix} x \\ y_n \\ z_n \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

– y axis

$$\begin{bmatrix} x_n \\ y \\ z_n \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

,

– z axis

$$\begin{bmatrix} x_n \\ y_n \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$