Computer Graphics

Two –Dimensional Transformations

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Two Dimensional Transformation

- In order to manipulate object in two dimensional space, we must apply various transformation functions to object.
- This allows us to change the position, size, and orientation of the objects.
- Transformations are used to position objects, to shape objects, to change viewing positions, and even to change how something is viewed.

Two Dimensional Transformation

- There are two complementary points of view for describing object movement.
  - The first is that the object itself is moved relative to a stationary coordinate system or background. The mathematical statement of this viewpoint is described by geometric transformations applied to each point of the object.
  - The second point of view holds that the object is held stationary while the coordinate system is moved relative to the object. This effect is attained through the application of coordinate transformations.

Two Dimensional Transformation

- An example involves the motion of an automobile against a scenic background.
  - We can simulate this by moving the automobile while keeping the backdrop fixed (a geometric transformation).
  - We can also keep the automobile fixed while moving the backdrop scenery (a coordinate transformation).

Geometric Transformations

- Let us impose a coordinate system on a plane.
- An object Obj in the plane can be considered as a set of points.
- Every object point P has coordinates (x, y), and so the object is the sum total of all its coordinate points.
- If the object is moved to a new position, it can be regarded as a new object Obj' , all of whose coordinate point P' can be obtained from the original points P by the application of a geometric transformation.

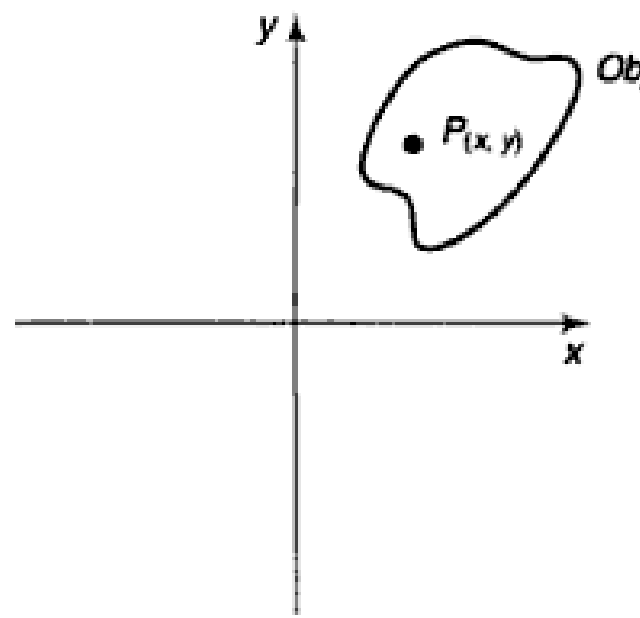
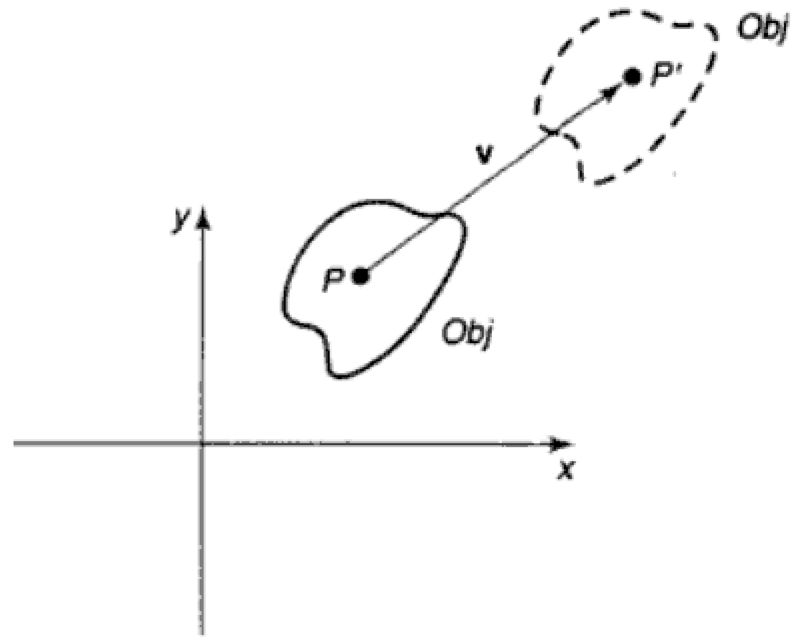


Fig. 4.1

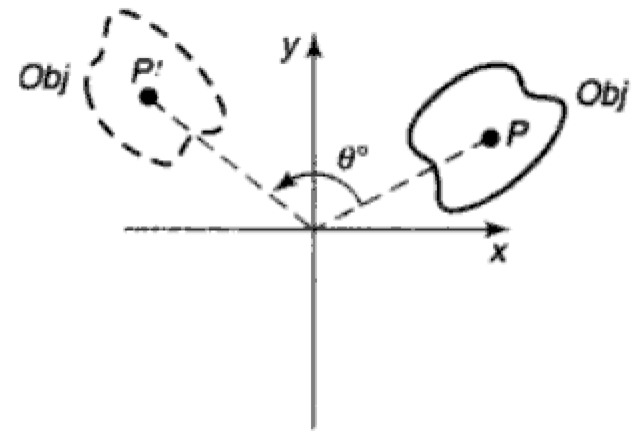
Translation

- In translation, an object is displaced a given distance and direction from its original position.
- If the displacement is given by the vector v= $t_x I + t_y$ j the new object point P'(x', y') can be found by applying the transformation T_v to P(x, y)
- $P' = T_v(P)$
- Where $x' = x + t_x$ and $y' = y + t_y$



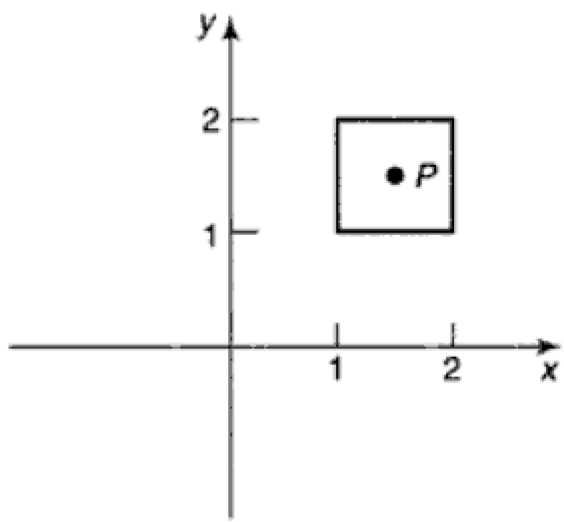
Rotation about the Origin

- In rotation, the object is rotated °θ about the origin.
- The convention is that the direction of rotation is counterclockwise if θ is a positive angle .
- and clockwise if θ is a negative angle.
- The transformation of rotation R_θ is $P' = R_\theta(P)$
- where $x' = x\cos(\theta) - y\sin(\theta)$ and $y' = x\sin(\theta) + y\cos(\theta)$

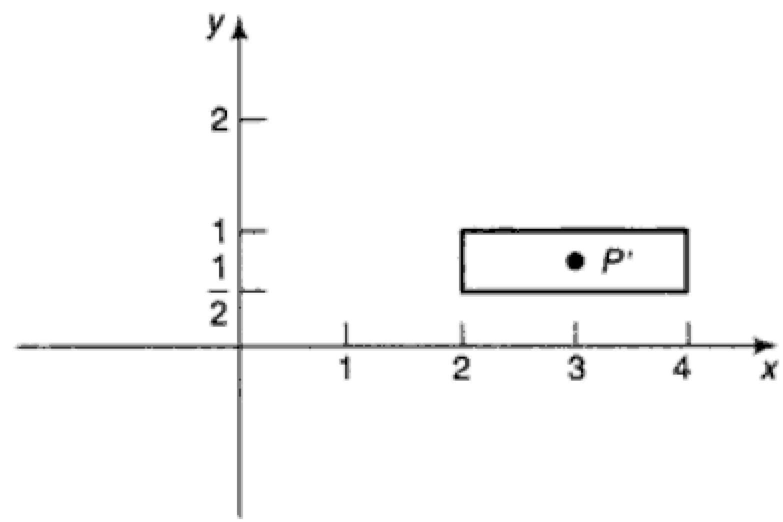


Scaling with Respect to the origin

- Scaling is the process of expanding or compressing the dimension of an object.
- Positive scaling constants s_x and s_y are used to describe changes in length with respect to the x direction and y direction, respectively.
- A scaling constant greater than one indicates an expansion of length, and less than one, compression of length.



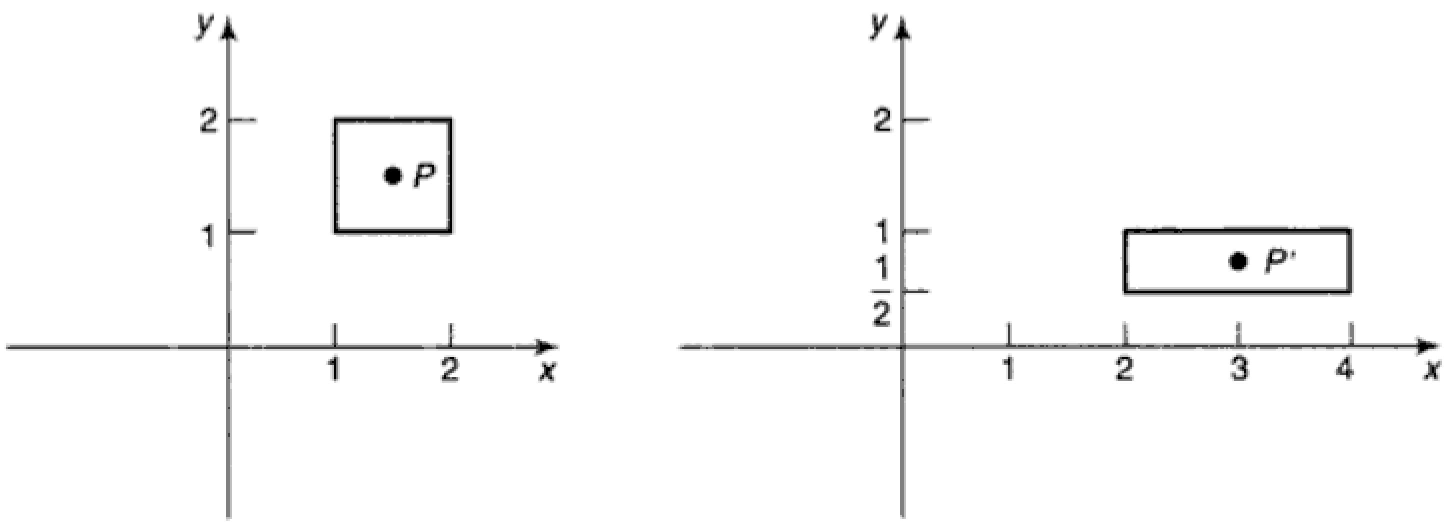
(a)



(b)

Scaling with Respect to the origin

- The scaling transformation $S_{Sx,Sy}$ is given by $P' = S_{Sx,Sy}(P)$
- where $x' = s_x x$ and $y' = s_y y$.
- If scaling constants have same value , its called homogeneous . if s>1, magnification & s<1 reduction
- After a scaling transformation is performed, the new object is located at a different position relative to the origin.
- In fac
  fixed



(a) Original Object

(b) Scaling factors $s_x = 2$
Scaling factors $s_y = 1/2$

Mirror Reflection about an Axis

- If either the x and y axis is treated as a mirror, the object has a mirror image or reflection.
- Since the reflection P’ of an object point P is located the same distance from the mirror as P (Fig. 4.5), the mirror reflection transformation M_x about the x-axis is given by $P' = M_x (P)$

  where $x' = x$ and $y' = -y$.
- Similarly, the mirror reflection about the y-axis is $P' = M_y (P) y$.

  where and $x' = -x-$ and $y' = y$.

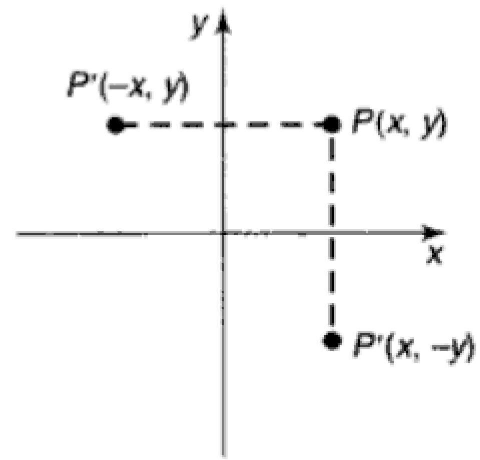


Fig. 4.5

Coordinate Transformations

Suppose that we have two coordinate systems in the plane. The first system is located at origin O and has coordinates axes xy. The second coordinate system is located at origin O' and has coordinate axes $x'y'$ (Fig. 4.6). Now each point in the plane has two coordinate descriptions: (x, y) or (x', y'), depending on which coordinate system is used. If we think of the second system $x'y'$ as arising from a transformation applied to the first system xy, we say that a *coordinate transformation* has been applied. We can describe this transformation by determining how the (x', y') coordinates of a point P are related to the (x, y) coordinates of the same point.

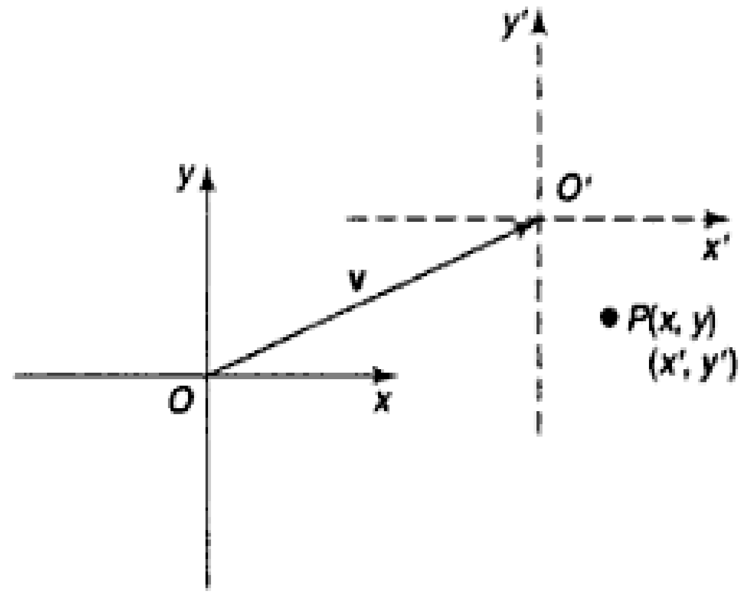


Fig. 4.6

Translation

If the xy coordinate system is displaced to a new position, where the direction and distance of the displacement is given by the vector $\mathbf{v} = t_x\mathbf{I} + t_y\mathbf{J}$, the coordinates of a point in both systems are related by the translation transformation $\bar{T}_{\mathbf{v}}$:

$$(x', y') = \bar{T}_{\mathbf{v}}(x, y)$$

where $x' = x - t_x$ and $y' = y - t_y$.

Rotation about the Origin

The xy system is rotated $\theta°$ about the origin (see Fig. 4.7). Then the coordinates of a point in both systems are related by the rotation transformation R_θ:

$$(x', y') = R_\theta(x, y)$$

where $x' = x\cos(\theta) + y\sin(\theta)$ and $y' = -x\sin(\theta) + y\cos(\theta)$.

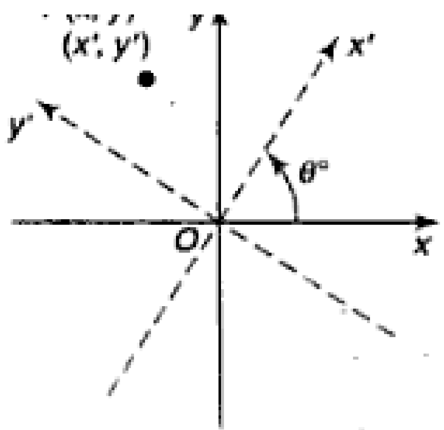


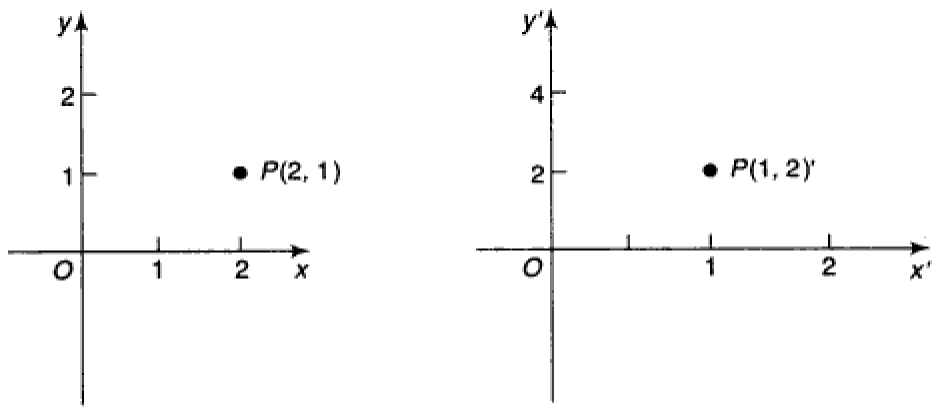
Fig.4.7

Scaling with Respect to the Origin

Suppose that a new coordinate system is formed by leaving the origin and coordinate axes unchanged, but introducing different units of measurement along the x and y axes. If the new units are obtained from the old units by a scaling of s_x along the x axis and s_y along the y axis, the coordinates in the new system are related to coordinates in the old system through the scaling transformation $\bar{S}_{s_x,s_y}$:

$$(x', y') = \bar{S}_{s_x,s_y}(x, y)$$

where $x' = (1/s_x)x$ and $y' = (1/s_y)y$. Figure 4.8 shows coordinate scaling transformation using scaling factors $s_x = 2$ and $s_y = \frac{1}{2}$.



(a) Old units (b) New units

Mirror Reflection about an Axis

If the new coordinate system is obtained by reflecting the old system about either x or y axis, the relationship between coordinates is given by the coordinate transformations $\bar{M}_x$ and $\bar{M}_y$. For reflection about the x axis [Fig. 4.9(a)]

$$(x', y') = \bar{M}_x(x, y)$$

where $x' = x$ and $y' = -y$. For reflection about the y axis (Fig. 4.9)

$$(x', y') = \bar{M}_y(x, y)$$

where $x' = -x$ and $y' = y$.

Notice that the reflected coordinate system is left-handed; thus reflection changes the orientation of the coordinate system. Also note that $\bar{M}_x = \bar{S}_{1,-1}$ and $\bar{M}_y = \bar{S}_{-1,1}$.

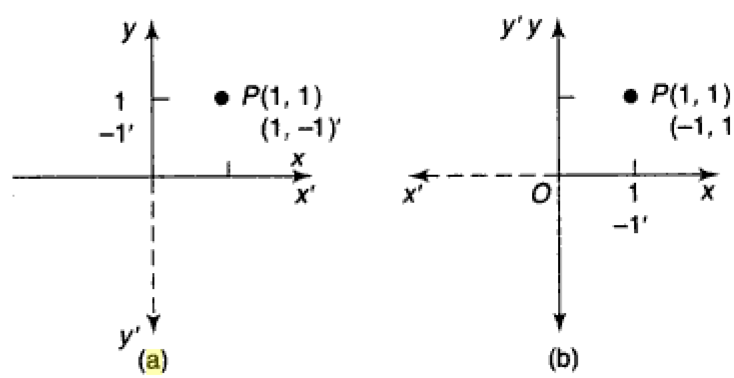


Fig. 4.9