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Research Article

A New Approach to Studying Net Present Value and the Internal Rate of Return of Engineering Projects under Uncertainty with Three-Dimensional Graphs

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Cost-benefit analysis (CBA) is very useful when appraising engineering projects and examining their long-term financial and social sustainability. However, the inherent uncertainty in the estimation of completion time, final costs, and the realization of benefits often act as an impediment to its application. Since the emergence of fuzzy set theory, there have been significant developments in uncertainty modelling in project evaluation and investment analysis, primarily in the area of formulating a fuzzy version of CBA. In this context, in studying the key indicators of CBA, whereas fuzzy net present value (fNPV) has been investigated quite extensively, there are significant issues in the calculation of fuzzy internal rate of return (fIRR) that have not been addressed. Hence, this paper presents a new conceptual model for studying and calculating fNPV and fIRR. Three-dimensional fNPV and fIRR graphs are introduced as a means of visualizing uncertainty. A new approach is presented for the precise calculation of fIRR. To facilitate practical application, a computerization process is also presented. Additionally, the proposed methodology is exemplified in a sample motorway project whereby its advantages over traditional stochastic uncertainty modelling techniques such as Monte Carlo analysis are discussed. Overall, it is concluded that the new approach is very promising for modelling uncertainty during project evaluation for both project managers and project stakeholders.

1. Introduction

Cost-benefit analysis (CBA) is a valuable decision support tool in project evaluation in both the public and private sectors [1]. It is widely acknowledged that the fundamental principles of CBA are accredited to the work of the French civil engineer and economist Jules Dupuit in the 1840s [2]. After being used systematically in the U.S. in the 1930s, by the end of the 1960s, the use of CBA spread around the world in both developed and developing countries [3]. Its broad purpose is to help decision-making and to make it more rational by the more efficient allocation of available resources [4]. Today, many international financial institutions and international organisations such as the European Investment Bank use CBA to appraise the economic desirability of projects [5].

Reasonably, the fundamental difficulties in the estimation of completion time, final costs, and the realization of benefits often act as an impediment to the application of CBA. Hence, besides its significance and importance, there are limitations to its application because of the underlying approximations, the working hypotheses, and the possible lack of data [6]. Additionally, Belli and Guerrero [7] conclude that when CBA project documents are assessed, risk analysis emerges as one of the weakest areas. Since uncertainty management in CBA is identified as problematic, research is needed to improve existing techniques. It is envisaged that innovations and improvements can increase its importance in engineering decision-making theory and practice.

The primary indicators in CBA are the net present value (NPV) which is expressed in monetary values and the internal rate of return (IRR) [3]. The purpose of this paper is to present an uncertainty management model that applies fuzzy set theory to these indicators. Also, an automation process based on computer processing is presented to facilitate application. Last but not least, a case study is discussed to exemplify both the application of the approach presented in this paper and the introduction of uncertainty due to decisions made during the design and planning phases of an engineering project. Finally, the overall conclusions of this work are presented.

2. Literature Review

2.1. Stochastic and Fuzzy Risk Assessment in CBA. Stochastic risk assessment, in CBA, can be performed primarily with Monte Carlo risk analysis, which is a sophisticated technique that starts by specifying stochastic probability distributions for significant uncertain quantitative assumptions. After that, a trial is taken by taking a random draw from the distribution of each parameter. This step is repeated several times in order to produce a histogram that depicts the realization of net benefits. The underlying assumption is that as the number of trials increases, the frequencies will converge towards the true underlying probabilities [4].

There has been extensive research in applying fuzzy set theory in CBA. Kaufmann and Gupta [8] discussed the discounting problem with fuzzy discounting rates and crisp (nonfuzzy) investment costs. Wang and Liang [9] proposed two algorithms to conduct CBA in a fuzzy environment in which it is difficult to obtain exact assessment data such as investment benefit, expenses, project lifetime, gross income, expenses, and depreciation. Mohamed and McCowan [10] proposed a method for modelling the effects of both monetary (construction cost and annual revenue) and nonmonetary (political, environmental, organizational, competition, and market share) aspects of investment options with possibility theory. Schjaer-Jacobsen [11] set out to examine the possibility of attaining a reasonably useful and realistic picture of the economic consequences of strategic decisions when little is known about the future. He argued that the quality of available information to decision-makers renders traditional decision theory and investment calculations obsolete, while he also demonstrated the representation of economic uncertainties in an investment example with the aid of triangular fuzzy numbers. Dompere [12] studied the discounting process under uncertainty and examined the theory of the fuzzy present value. Chiu and Park [13] developed a fuzzy cash flow analysis for engineering decisions. Sorenson and Lavelle [14] compared fuzzy set and probabilistic paradigms for ranking vague economic investment's information and concluded that cash flows and interest rates should be modelled by fuzzy sets and ranked with a fuzzy ranking method. Sewastjanow and Dymowa [15] recognized how the obtaining of fuzzy IRR is a rather open problem and to this extent examined a framework for solving fuzzy equations. Tsao [16] presented a series of algorithms to calculate fuzzy net present values of capital investments in an environment with uncertainty. He suggested that the imprecision and uncertainty of the project cash flow are higher than that of the cost of capital.

Beyond the development of a fuzzy version of CBA, there is significant research in applying fuzzy set theory in uncertainty in variables that regard the costs and the cash flow of projects. Kishk [17] applied fuzzy set theory in a whole life costing modelling. Shaheen et al. [18] presented a methodology for extracting fuzzy numbers from experts and processing the information in fuzzy range cost estimation analysis. More so, fuzzy project scheduling (FPS) is based on the application fuzzy set theory in traditional scheduling techniques and is useful in dealing with circumstances involving uncertainty, imprecision, vagueness, and incomplete data [19]. The critical concept is modelling activity duration and cost with fuzzy numbers and thereby calculating project duration and cost, activity start and finish dates, and activity criticality. As such, Maravas and Pantouvakis [20] have shown how fuzzy cost estimates of project activities can be combined with fuzzy project scheduling to yield project cash flow projections.

Regarding project benefits, in the specific case of transportation projects, fuzzy traffic assignment models indicate the region of the expected project benefits. Teodorovic [21] emphasized the importance of fuzzy logic systems as universal approximators in solving traffic and transportation problems. Henn and Ottomanelli [22] applied possibility theory in traffic assignment modelling. Ghatee and Hashemi [23] proposed a traffic assignment model with a fuzzy level of demand. Triangular fuzzy numbers were used to show the imprecise number of travellers who want to travel between origin-destination pairs. Caggiani et al. [24] used fuzzy programming to improve origin-destination matrix estimation based on traffic counts and other uncertain data. De Ona et al. [25] used fuzzy optimization to obtain adjusted values of field traffic volume data to meet consistency constraints.

While the NPV and IRR are the most widespread and accepted indicators when conducting CBA analysis, there are significant developments in the study of IRR. As such, Magni [26] introduced the concept of the average internal rate of return (AIRR) as an alternative to the well-established IRR. While dismissing the IRR equation, he argued about the superiority of the AIRR. Guerra et al. [27] applied fuzzy set theory to the AIRR to study investment appraisal under uncertainty. Jiang [28] presented a particular case of a continuous AIRR, named excess return of time-scaled contributions (ERTC) that can be used in capital budgeting and project finance. Mørch et al. [29] considered the maximization of the AIRR in the renewal of maritime shipping capacity.

Overall, besides the significant research in applying fuzzy set theory to CBA, there are significant issues that need to be researched—primarily, the study of the variation of fuzzy NPV in regard to the discount rate and the calculation of fuzzy IRR. Additionally, the emergence of new fuzzy techniques in cost estimations, cash flow prediction, and benefit analysis provides an opportunity for formulating fuzzy variables that can thereafter be used as base estimates in a holistic risk assessment methodology. Finally, the newly introduced AIRR and its fuzzy equivalent could potentially be adopted in CBA analysis.

2.2. Fundamentals of Fuzzy Set Theory. Fuzzy Set Theory is used to describe and quantify uncertainty and imprecision in data and functional relationships. A fuzzy subset A of a universe of discourse U is characterized by a membership function μ_A: U $\rightarrow$ [0, 1] which associates with each element x of U a number $\mu_A(x)$ in the interval [0, 1] which represents the grade of membership of x in A. In fuzzy set theory, the triangular membership function which is defined by three numbers a, b, and c is encountered very often. Hence, a triangular fuzzy number $\tilde{x} = \langle a, b, c \rangle$ has the following membership function:

$$\mu_A(x) = \begin{cases} 0, & x < a, \\ \frac{(x-a)}{(b-a)}, & a \leq x \leq b, \\ \frac{(c-x)}{(c-b)}, & b \leq x \leq c, \\ 0, & x > c. \end{cases} \quad (1)$$

Every fuzzy set A can be associated to a collection of crisp sets known as α-cuts (alpha-cuts) or α-level sets. An α-cut is a crisp set consisting of elements of A which belong to the fuzzy set at least to a degree of α. As such, if A is a subset of a universe U, then an α-level set of A is a nonfuzzy set denoted by A_α which comprises all elements of U whose grade membership in A is greater than or equal to α [30]. In symbols,

$$A_\alpha = \{u \mid \mu_A(u) \geq \alpha\}, \quad (2)$$

where α is a parameter in the range $0 < \alpha \leq 1$.

Effectively, an α-cut is a means to defuzzify a fuzzy set into a crisp set at desired α-levels which reflect the perceived risk. More specifically, every α-cut indicates the pessimistic and optimistic values for the same risk level, and in the case of a triangular fuzzy number, it is given by the following formula:

$$A_\alpha = [\alpha(b-a) + a, \ \alpha(b-c) + c]. \quad (3)$$

In many cases, it is necessary to compare fuzzy numbers in order to attain a linear ordering. In such cases, the removal number can be defined as the first criterion for the linear ordering. Essentially, it is an ordinary representative of the fuzzy number; in the case of a triangular fuzzy number, it is given by the following formula [8]:

$$u = \frac{1}{4(a + 2b + c)}. \quad (4)$$

The second criterion is the mode of the fuzzy number, that is, "b" for the triangular fuzzy number $\tilde{x} = \langle a, b, c \rangle$. Finally, the divergence of the fuzzy number around the mode is the third criterion. It is given by the following formula:

$$\mathrm{div}_x = c - a. \quad (5)$$

3. Fuzzy NPV and IRR

3.1. Three-Dimensional Graphical Representation of Fuzzy NPV and IRR. The net present value (NPV) is essentially the discounted net cash flow—the sum that results when the discounted expected financial costs of investment are subtracted from the discounted value of the expected benefits:

$$\mathrm{NPV} = C_0 + \frac{C_1}{1+i} + \frac{C_2}{(1+i)^2} + \cdots + \frac{C_n}{(1+i)^n}, \quad (6)$$

where i is the crisp financial discount rate and C_n is the crisp net cash flow at period n.

However, in the presence of uncertainty, all values may be modelled with fuzzy numbers. Hence, the fuzzy-net present value (fNPV) is defined as follows [31]:

$$\mathrm{fNPV} = \tilde{C}_0 + \frac{\tilde{C}_1}{1+\tilde{i}} + \frac{\tilde{C}_2}{(1+\tilde{i})^2} + \cdots + \frac{\tilde{C}_n}{(1+\tilde{i})^n}, \quad (7)$$

where $\tilde{i}$ is the fuzzy financial discount rate and $\tilde{C}_n$ is the fuzzy net cash flow at period n.

The crisp internal rate of return (IRR) is defined as the discount rate for which the net present value is equal to zero. In essence, it is the discount rate for which the costs are equal to the benefits. It is a measure of the profitability and the final yield of the investment. Thus, in practice, a project is more desirable if it has a higher value of IRR. However, the IRR cannot be calculated analytically from Equation (6). To this extent, numerical methods are employed to find an acceptable value based on convergence criteria. Since the fuzzy net present value (fNPV) is a fuzzy variable, the fuzzy internal rate of return (fIRR) is expected to be a set of discount rates for which fNPV is equal to zero instead of a single number. Hence, the calculation of this fIRR poses a significant challenge.

To this purpose, it is proposed that Cartesian geometry and a three-dimensional Euclidean space are used to graph fNPV and calculate fIRR. Thus, uncertainty can be represented by a three-dimensional plot in which the x axis represents the discount rate, the y axis the NPV, and the z axis the value of possibility [32]. In effect, these plots give the ability to scan across various discount rates and show the membership functions of fNPV for every such value (Figure 1). The plots can show the change in the uncertainty of fNPV in regard to the value of the discount rate. Thus, examining the slope of this plot and the change of the width of individual fNPV shows the variation of uncertainty in the project.

Besides, gaining insights into fNPV, three-dimensional graphs provide a novel manner for defining and calculating fIRR. Hence, the fuzzy internal rate of return (fIRR) is the set that is defined by the intersection of the xz plane (x and z axes at $y = 0$) with the fuzzy net present value (fNPV) for various values of the discount rate:

$$\mu_{\mathrm{fIRR}} = P_{xz} \cap \mu_{\underset{i=0}{\overset{R}{\mathrm{fNPV}}}}, \quad (8)$$

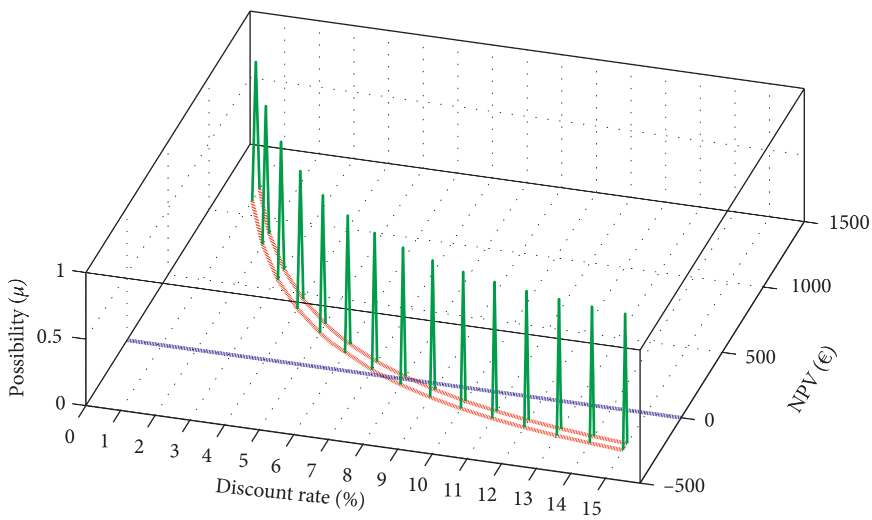

FIGURE 1: Three-dimensional fNPV graph.

where μfIRR is the membership function of the fuzzy internal rate of return, P_{xz} is the xz plane that is determined by the x and z axis at $y = 0$, and $\mu_{\mathrm{fNPV}}{}_{i=0}^{R}$ are all the membership functions of the fuzzy net present value from the discount rate of 0 to a selected value of R.

An example of such a three-dimensional plot is provided in Figure 2, in which five fNPV membership functions are plotted for their respective discount rates. The intersection of these five fNPV membership functions with P_{xz} ($y = 0$) defines 5 points which provide an estimate for fIRR. In effect, 5 points of the fIRR are calculated for the given interest rates (7.4%, 7.6%, 7.8%, 8%, and 8.26%).

3.2. Computerization Process. In order to get the exact shape of fNPV and fIRR, mathematical operations must be performed on the α-level sets of the variables of CBA with the aid of a computer program. Hence, if a triangular fuzzy number is studied at 0.1 interval α-cuts with Equation (3), as seen in Figure 3, it is represented by 21 numbers: 10 for the optimistic values, 10 for the pessimistic, and 1 for the value of no uncertainty ($\alpha = 1$). Thereafter, there are two steps.

Step 1: calculate fNPV for a range of discount rates. The fuzzy cash flow is calculated based on the underlying fuzzy variables. Then, operations on fuzzy numbers are performed on the corresponding α-cut values between them. This process is repeated for a range of discount rates (from 0 to R) to calculate the respective membership functions for the fNPV.

Step 2: calculate fIRR based on the planar intersection with fNPV. The program searches to find the specific values of the discount rate for which a specific fNPV membership function crosses P_{xz} ($y = 0$). Then, it loops over all the points of the α-cuts in order to find the two points in which there is a change of sign of the value of NPV. In Figure 3, we see that the intersection of fNPV with P_{xz} is between the two points of the α-cut levels of 0.3 and 0.4. The precise coordinates of the point of the intersection are calculated using the dot product formula for P_{xz} and the two other respective points. Understandably, the numerical precision of the calculation of fIRR depends on the increments of the discount rate as well as the selected number of α-cuts.

4. Case Study

In order to demonstrate the application of the model to civil engineering projects, it is applied to the data of a case study that is presented in the "Guide to Cost-Benefit Analysis of Investment Projects (Economic appraisal tool for Cohesion Policy 2014–2020)" of the European Commission [33]. The project concerns the construction of a new 16.4 km tolled motorway which is part of the Trans-European Network for Transport. The motorway will reduce traffic on an existing road which carries annual daily traffic of more than 18,000 vehicles. The motorway has 2×2 lanes (plus an emergency lane) with a width of 27.5 m, 3 junctions, 3 bridges (total length 2,200 m), 4 overpasses (total length 800 m), and 1 tunnel with two tubes (length 2,200 m). The socioeconomic analysis includes following monetised benefits, which are consistent with the project objectives, that are, faster travel on a safer road with separated carriageways, travel time savings, vehicle operating cost savings, environmental savings (CO_2 reduction), and accident cost savings [33]. Investment costs include the construction cost, whereas operating costs are the sum of maintenance costs with general expenses. During years 15–19, there are significant rehabilitation and renewal works on the motorway. At the end of the valuation period (30 years), the infrastructure retains a residual value. The resulting cash flows are shown in Table 1.

Hence, in socioeconomic analysis, the fuzzy net cash flow of Equation (7) is calculated as the difference between social benefits and costs:

$$\widetilde{C}_n = \mathrm{T\widetilde{S}}_n + \mathrm{V\widetilde{O}C}_n + \widetilde{A}_n + \widetilde{E}_n - \mathrm{I\widetilde{C}}_n - \mathrm{O\widetilde{M}}_n, \qquad (9)$$

where $\mathrm{T\widetilde{S}}_n$ is the time savings, $\mathrm{V\widetilde{O}C}_n$ is the vehicle operating costs savings, $\widetilde{A}_n$ is the accident savings, $\widetilde{E}_n$ is the

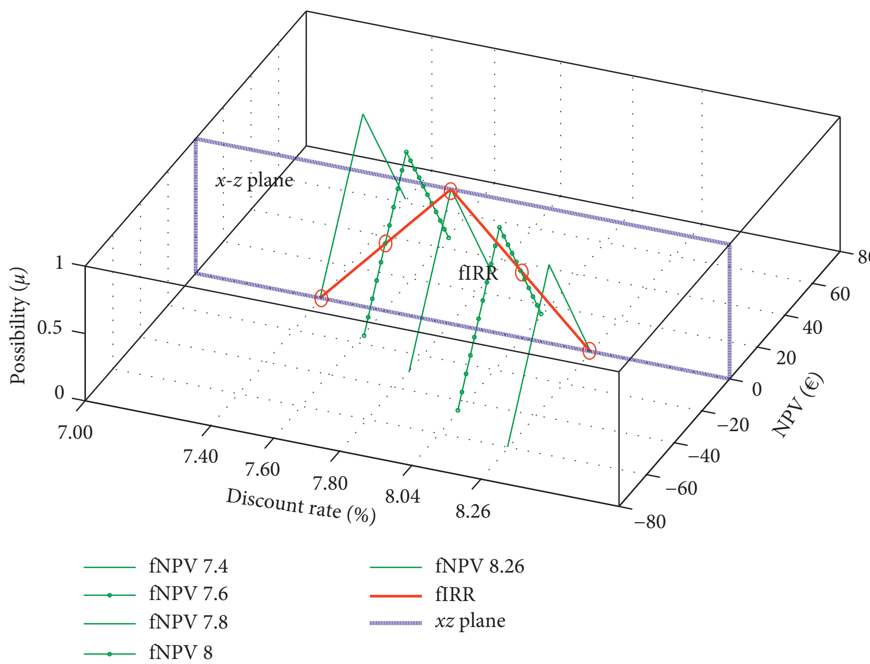

Figure 2: Definition and calculation of fIRR.

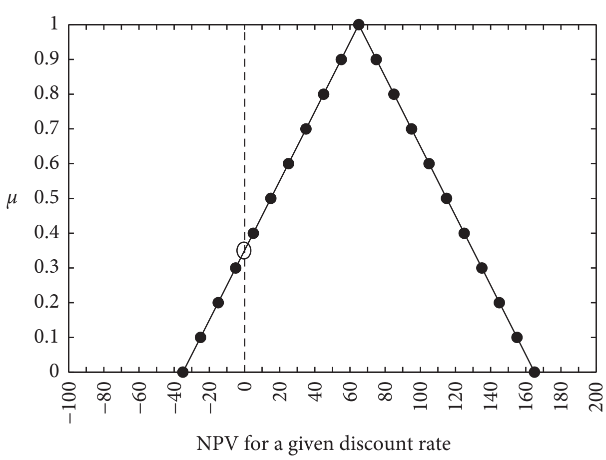

Figure 3: α-level sets of fNPV.

environmental savings (CO_2 reduction), $\widetilde{IC}_n$ is the investment cost, and $\widetilde{OM}_n$ is the operation and maintenance cost.

The fundamental concept is to model the uncertainty in all relevant parameters in the analysis of the cash flow as fuzzy numbers. As a result, the indices of the analysis will also be fuzzy numbers and consequently provide a basis for risk assessment. A sensitivity analysis reveals that the investment cost and the time savings are the most critical variables [33]. Thus, the risk analysis will study the scenario that the investment cost may be reduced by 5% or increased by 20%, whereas the time savings may be reduced by 30% or may be increased by 15%. Thus, the baseline variables of investment cost and operation and maintenance costs (in Table 1) are fuzzified by providing the optimistic and pessimistic values based on a percentage change valuation. Hence, for instance, a −5% and +20% uncertainty regarding uncertainty in the investment cost of the first year is represented as $\widetilde{IC}_1 = \langle -113.9, -94.9, -90.16\rangle$, whereas as −30% and +15% uncertainty regarding the time savings in the 4th year are $\widetilde{TS}_4 = \langle 7.49, 10.7, 12.31\rangle$. Crisp (nonfuzzy) operation and maintenance costs in the 4th year are represented as $\widetilde{OM}_4 = \langle -0.8, -0.8, -0.8\rangle$. Overall, costs are denoted with a negative sign, whereas benefits with a positive sign.

The model aims to find the robustness of fNPV and fIRR in regard to fluctuations in time benefits and investment cost. Additionally, in order to interpret the graphs of fuzzy variables at different α-cuts which represent different levels of possibility, the following rough assumptions can be made: $\alpha = 1$ corresponds to no risk, $\alpha = 0.7$ to low risk, $\alpha = 0.5$ to medium risk, $\alpha = 0.3$ to high risk, and $\alpha = 0+$ to the most extreme scenario. Thus, in Figure 4, under deterministic analysis ($\alpha = 1$), the NPV would be 86 mil. € when considering a discount rate of 5%. However, in the absolute worst case which occurs with the simultaneous increase of investment costs and the reduction of time savings, the fNPV is −36 mil €. Similarly, in the best case, the fNPV will be 136 mil. € with a reduction of investment costs and an increase in time savings. The unevenness of the baseline variables creates the asymmetry in the graph.

Figure 5 plots the fNPV in three dimensions when the discount rate varies from 0% to 15%. From the graph, it can be seen that the values of fNPV approach zero near a discount rate of 4 to 9%. As such, it is imperative to examine this region in greater detail. Another interesting observation is that, at $r = 0\%$, the NPV varies between 342 and 687 mil. €. However, as the discount rate is increased, the difference between the optimistic and pessimistic scenarios is reduced dramatically, that is, at $r = 15\%$, the difference is only 80 mil.

Table 1: Baseline data for motorway construction [33].

Year	Costs (mil. €)		Benefits (mil. €)			
	IC_n	OM_n	TS_n	VOC_n	A_n	E_n
1	(94.9)	0	0	0	0	0
2	(92.1)	0	0	0	0	0
3	(57)	0	0	0	0	0
4	0	(0.8)	10.7	1.3	0.4	0.1
5	0	(0.8)	11.5	1.4	0.4	0.1
6	0	(0.8)	12.3	1.5	0.5	0.1
7	0	(0.8)	13.2	1.5	0.5	0.1
8	0	(0.8)	14.1	1.6	0.5	0.1
9	0	(0.8)	15	1.7	0.5	0.2
10	0	(0.8)	16	1.8	0.6	0.2
11	0	(0.8)	17	1.9	0.6	0.2
12	0	(0.8)	18	2	0.6	0.2
13	0	(0.8)	19	2	0.7	0.2
14	0	(0.8)	20	2.1	0.7	0.2
15	0	(6.9)	20.7	2.1	0.7	0.2
16	0	(6.2)	22	2.2	0.7	0.2
17	0	(5.8)	23	2.2	0.8	0.2
18	0	(5.2)	24	2.3	0.8	0.3
19	0	(4.4)	25	2.4	0.8	0.3
20	0	(0.8)	25.4	2.4	0.9	0.3
21	0	(0.8)	26	2.5	0.9	0.3
22	0	(0.8)	29	2.5	1	0.3
23	0	(0.8)	29	2.6	1	0.4
24	0	(0.8)	30	2.6	1	0.4
25	0	(0.8)	30.5	2.7	1	0.4
26	0	(0.8)	34	2.8	1.1	0.4
27	0	(0.8)	35	2.8	1.2	0.4
28	0	(0.8)	36	2.9	1.2	0.5
29	0	(0.8)	37	2.9	1.2	0.5
30	151	(0.9)	37.7	3	1.2	0.5

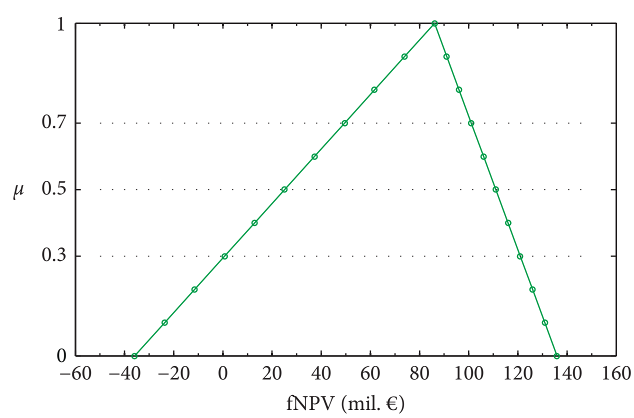

Figure 4: fNPV for a discount rate of 5%.

€. Eventually, the optimistic and pessimistic scenarios both yield negative values for NPV.

Figure 6 provides a closer insight as to how the fIRR can be formed by the intersection of P_{xz} (at $y = 0$) with the fuzzy sets of fNPV. It shows 25 NPV membership function graphs that correspond to discount rates from 4% to 8.8% at 0.2 increments. The computer program calculates the intersection of every NPV membership function with P_{xz}. Then, the fIRR is formed by connecting 22 points since 3 membership functions do not intersect with P_{xz}. Understandably, the fIRR will be represented with higher resolution if the discount rate increments are at 0.1 or lower.

Figure 7 shows a two-dimensional plot of the three-dimensional graph as if it were seen from an "aerial view" above the z axis. It is apparent that the spread of the NPV membership functions gets narrower as the discount rate increases. It is also evident how the fIRR is formed from the intersection of various fNPV membership functions with P_{xz} (NPV = 0). Finally, it can be seen how 3 fNPV membership functions for the discount rates of 4%, 8.6%, and 8.8 % do not intersect with P_{xz}.

The methodology has succeeded in calculating fIRR. Thus, Figure 8 provides a two-dimensional plot of the fIRR that was calculated previously. Essentially, the crisp value of IRR is 7.2% but because of the presence of uncertainty, it varies from 4.2% to 8.4%. Hence, there is a significant possibility that the outcome of the project will be significantly lower or somewhat higher than what initially expected.

5. Discussion

5.1. Interpretation of Results. The ability to plot and study fNPV in three dimensions is very promising. It is now possible to see how fNPV and fIRR are related to the discount rate. Hence, alternative projects should not only be compared based solely on a single fNPV but also on the way the fNPV changes in regard to the discount rate. Also, the fIRR can be calculated in an intuitive and very effective manner. As expected, there is not a single value of a discount rate that sets NPV equal to zero. Overall, negative values of NPV or low values for IRR indicate that there are significant risks in the project. Hence, risk control strategies should be employed, or funds should be directed to alternative investments.

In the case study, the uncertainty analysis reveals the problems stemming from the inability to control construction cost and estimate the future benefits. Individually, the tunnel costs 80 mil. € or approximately one-third of the total construction cost. Even though there are several geological studies, it may not be possible by the project manager to control the cost escalation arising from adverse geological conditions that can't be predicted [33]. Also, it is unlikely that the original estimation of benefits will be correct due to the inherent volatility of traffic demand.

Decision-maker's choice depends largely on his their attitude towards risk [34]. Decision-makers should have the best tools available to understand and comprehend the risks they are undertaking. Thus, they should be able to understand the relation of the input variables to the project outcome. Also, it is desirable that they can compare different projects to each other. As such, the fuzzy project performance indicators (fNPV and fIRR) of alternative projects can be compared with Equations (3)–(5).

5.2. Comparison to Stochastic Analysis. It is very interesting to compare the methodology for uncertainty modelling with

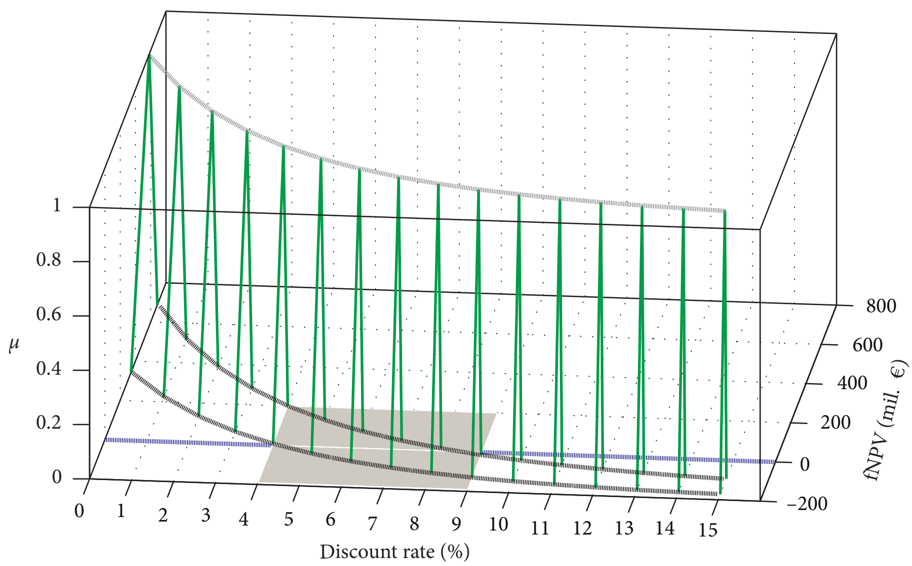

Figure 5: 3D plot of fNPV for various discount rates.

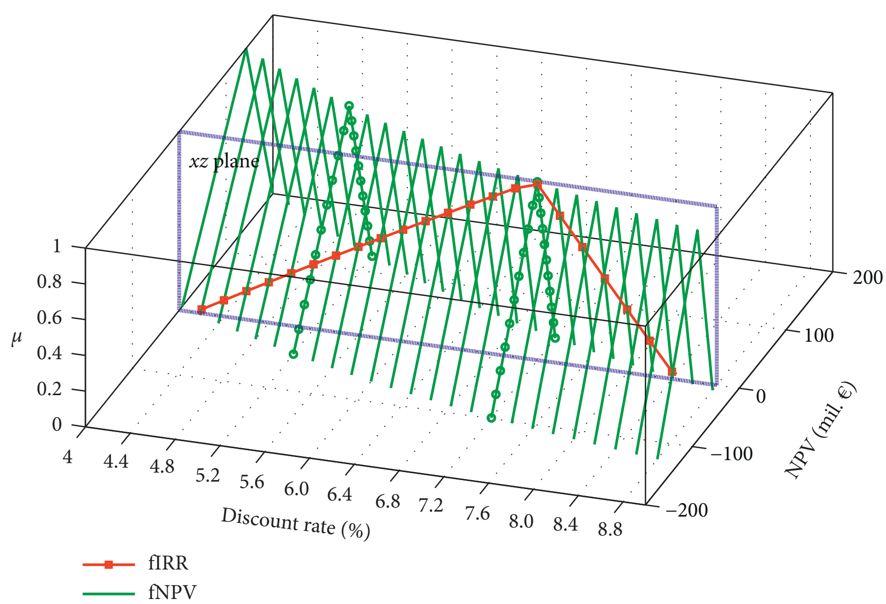

Figure 6: Calculation of fIRR.

fuzzy set theory to that of stochastic analysis with Monte Carlo analysis. Overall, the fuzzy methodology presents the following advantages: (a) computational efficiency: the results of fNPV and fIRR are derived through a single analytical calculation, contrary to Monte Carlo analysis that requires thousands of iterations; (b) repeatability: in Monte Carlo analysis, the results of every simulation are moderately different from each other due to the randomness of numbers that are generated by computers. On the other hand, for a given set of input variables, the results of a fuzzy analysis are 100% repeatable; and (c) uncertainty aggregation: project stakeholders may often need to assess the total risk of a group of projects or that of a programme. With fuzzy set theory, it is very straightforward to add indicators from many projects and thus attain an aggregated fuzzy performance indicator—something which is not feasible with probability distributions.

5.3. Fuzzy AIRR. A fascinating research question is if the proposed Euclidean space can be useful in studying the AIRR or its fuzzy equivalent. Since the AIRR is defined as the ratio of total profit to the total capital invested and it is calculated by dismissing the IRR equation, the Euclidean space does not present an advantage in its calculation which can be done quickly. However, in examining the graphs of

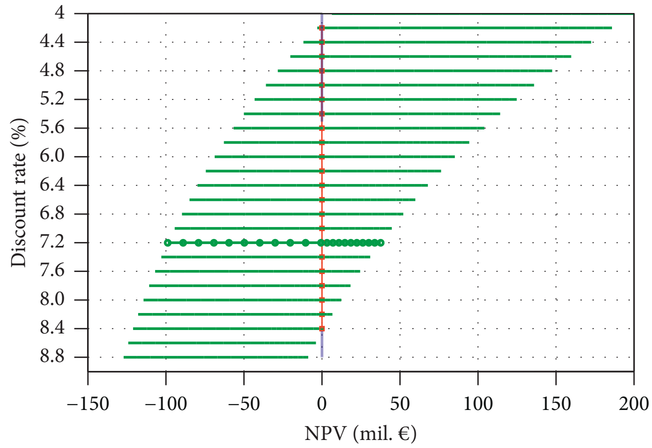

Figure 7: Aerial view of 3D fNPV graph for the motorway.

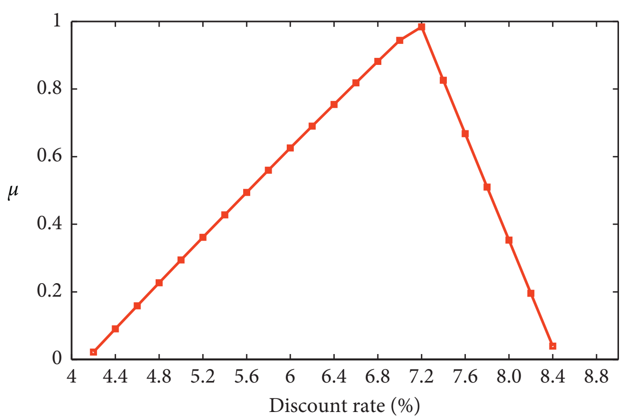

Figure 8: 2D plot of fIRR.

fuzzy NPV and AIRR, there may be considerable advantages in studying their relationship in the Euclidean space. Specifically, Magni [35] proposed the concept of an "isovalue line" which is essentially an indifference curve that supplies the same NPV on a two-dimensional graph of the AIRR vs. capital invested. Potentially, the concept of the "isovalue line" could be extended to that of an "isovalue surface" if a three-dimensional graph is employed (x axis: invested capital, y axis: AIRR, and z axis: possibility). The graphical tool could significantly enhance intuition and understanding of risks in projects. Thus, future research may be targeted towards studying fuzzy AIRR with three-dimensional graphs.

6. Conclusions

Using a Euclidean space, this paper presents an alternative approach to studying uncertainty in NPV and IRR based on fuzzy set theory. Its assumptions are realistic and intuitive for engineers; it has low mathematical complexity and is much simpler to computerize than stochastic risk analysis. With three-dimensional fNPV and fIRR graphs, project evaluation and selection can be seen for the first time from a different perspective. Also, the paper has succeeded in formulating a novel approach to calculating fIRR. These tools may help in examining projects with greater scrutiny to determine the robustness of the decision indices.

Finally, future research can be directed in formulating a holistic methodology that incorporates fIRR and fNPV with fuzzy project scheduling, fuzzy cash flow analysis, and benefits realization in an advanced management system. Additionally, the use of three-dimensional graphs may potentially increase the understanding of the newly introduced AIRR. Most importantly, fuzzy risk analysis should be seen as a core methodology that when coupled with other emerging techniques can increase the quality and realism of estimation data leading to the advanced management of projects.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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