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IRR performs better than NPV: A critical analysis of cases of multiple IRR and mutually exclusive and independent investments[1]

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Abstract:

This paper presents evidence to identify the appropriate investment criterion (IRR vs NPV) with emphasis on the controversial reinvestment assumption, multiple, negative, zero and no IRR, mutually exclusive investment and independent projects. The analysis is based on the estimated return of capital (ROC), return on invested capital (ROIC), capital amortization schedule (CAS) and the modified CAS (MCAS). The salient findings are:

a. There is no reinvestment of intermediate income in normal NCF investments. Incidences of reinvestment is evident in some of the NNCF investments where the opening balance (OB) in the CAS is positive. The problem of reinvestment at IRR or at hurdle rate and the resultant multiple IRR are the symptoms of the problem associated with some non-normal NCF (NNCF) investments.
b. NNCF with reinvestment of intermediate income always leads to multiple IRR. MCAS method eliminates the reinvestment income from the NNCF data and resolves the problem of multiple IRR and leads to unique IRR.
c. There is no IRR when there is no investment in zero year or investment year or there is no negative cash flow and no sign change. Negative IRR is common when the sum of NNCF is negative that and reveals capital loss (ROC not fully recovered). When the sum of NNCF is zero, the IRR will be zero.

[2] The opinions expressed in this article are those of the author(s) based on private research, and they do not reflect in any way those of the institutions to which the author(s) is affiliated.

Zero IRR indicates that the NNCF is sufficient to recover the capital cost (ROC) and ROIC not recovered.

d. In the case of mutually exclusive projects, selecting projects based on NPV at hurdle rate is misleading. A project selected based on higher NPV at hurdle rate ends-up with negative NPV when discounted by the higher IRR achieved by the counterpart project (with lower NPV). In the case of counterpart project, the NPV is not negative but zero at that higher IRR as the discount rate. This weakness of NPV is not exposed as NPV is a static point estimate (at hurdle rate). The NPV is not an appropriate criterion.

e. Among a range of discount rates tried, IRR is the only rate that indicates the full utilization of the NCF (makes the NPV zero) and therefore performs better than the NPV to select or rank mutually exclusive projects. NPV is the unutilized NCF and a static point estimate. NPV therefore provides incomplete information at hurdle rate. It fails to indicate the full ROIC that the Net cash flow (NCF) could support. NPV at hurdle rate can at best indicate whether the NCF is fully utilized (NPV = 0) or not fully utilized (positive NPV) or not adequate to cover the cost of capital (negative NPV), respectively. IRR, in contrast, explicitly indicates that the NCF is fully utilized (NPV and the closing balance (CB) in CAS are '0') and the ROIC plus ROC are fully recovered.

f. The modified IRR (MIRR) is not a solution to the problem of multiple IRR. MIRR assumes and includes reinvestment income without any evidence of reinvestment and therefore MIRR is a spurious estimate. The problem of reinvestment and the resultant multiple IRR is resolved by eliminating the reinvestment income under MCAS method and therefore IRR is unique.

These results consistently support the conclusion that IRR is the best criterion to accept or reject or rank mutually exclusive projects as well as independent projects. NPV will still be useful in other areas to estimate the present value. Authors of finance and economic textbooks and other publications may wish to review and consider appropriately update the sections on CBA or capital investment or capital budgeting.

1. Introduction:

Globally, there is growing interest in applying cost-benefit analysis (CBA) in investment analysis, valuation and in regulatory reforms. There are, however, continuing arguments on the superiority or inferiority of the two criteria, internal rate of return (IRR) and net present value (NPV). Survey of corporate managers has consistently shown that managers rank IRR ahead of NPV (Bhattacharyya, 2004). Companies in Germany and Netherlands preferred NPV, whereas IRR in UK and France (Brounen, De Jong and Koedijk, 2004). Brealey et al. (2009) commented that for many companies discounted cash flow (DCF) means IRR, not NPV. Private sector, large corporations and banks, use both criteria but prefer IRR because of use of comparison with the cost of capital (Osborne, 2010). Several academic papers preferred NPV over IRR as if they are not interrelated criteria. Both NPV and IRR are estimated by DCF method, using the same net cash flow (NCF) data and they are mathematically interrelated (IRR is the discount rate that makes NPV Zero). Brigham and Houston (2016) made a guess that if a survey were to be taken as of 2015, NPV would be at the top of this list.

The problem of multiple or negative or no IRR universally regarded as a fatal flaw for the IRR method (see Hazen, 2003). The prevailing conventional wisdom is that the IRR is not suitable for ranking and selecting mutually exclusive projects (for e.g. Osborne 2010, Kierulff 2012, World Bank 2013, Bora, 2015). Analysts and decision makers, therefore, opt for NPV as the better criterion to accept or reject projects or rank mutually exclusive projects. Another source of criticism is that IRR is not the return on the total capital but on the remaining capital or the declining balance of the investment (Kelleher and MacCormack 2004, Rich and Rose 2015, Dudley 1972). This problem can be easily resolved by estimating IRR at two levels.

Given the problems discussed above, it is important to investigate the validity of various controversies surrounding the CBA, investment analysis and capital budgeting so that analysts and decision makers can confidently apply these criteria in real life situations.

The purpose of this paper is therefore:

a. to provide a better insight into the conceptual issues relating to the reinvestment of intermediate income at IRR or at hurdle rate in NPV, the cases of multiple, negative and no IRR, ranking of mutually exclusive investments and return on total capital vs balance of capital; and
b. to recommend an appropriate criterion, based on analytical evidence, for all situations including mutually exclusive investments and multiple IRR or no IRR.

2. Literature Review:

Currently, IRR and NPV is being estimated using the net cash flow (NCF) data and the discounted cash flow (DCF) method. Arjunan (2017a) introduced a new method to estimate IRR and NPV which is based on capital amortization schedule (CAS). The CAS method reveals that reinvestment of intermediate income is associated with some of the NNCF that leads to multiple IRR. Arjunan (2017b) also introduced a modified CAS method (MCAS) that eliminates the problem of multiple IRR and leads to a unique IRR. When the reinvestment income is excluded from the NCF, there is no multiple IRR. These new methods are more transparent and exposes the strength and weakness of the IRR and NPV criteria.

White and VanLandingham (2015) reported that an analysis of the CBA studies completed by states in the USA shows that most lack some recommended technical features of rigorous CBA, but these reports are having a reported impact on state policy and budget decisions. Weber (2014) reported that the IRR is generally considered inferior to the net present value (NPV) as a tool for evaluating and ranking projects. World Bank (2013), in their Investment Project Financing guidance note indicated that the IRR is not suitable for the ranking of competing projects. Cuthbert and Cuthbert (2011) argued that IRR is potentially misleading. Some others concluded that IRR is a limited decision tool and advised the financial analysts to use it with caution (see also Kierulff 2012; Kelleher and MacCormack 2004). Phalippou (2009) stated that IRR has its own shortcomings and biases and creates room for managers to manipulate the performance reporting. Park (2005) recommended to abandon the IRR analysis and use the NPV criterion whenever analysts encounter multiple rates of return.

Some authors reported that IRR and NPV together guarantee the making of relevant decisions (Juhász, 2011, Hall and Millard, 2010). Berkovitch and Israel (2004) showed that the use of NPV as an investment criterion leads to inefficient capital budgeting outcomes and therefore other capital budgeting criteria, like the IRR and the profitability index (PI) will continue to dominate. Hazen, 2003, reported that the problem of multiple or nonexistent IRR is not really a flaw at all, and can be easily dealt with conceptually and procedurally. Several authors are of the view that multiple internal rates constitute a severe drawback and not helpful (Brealey and Myers 1996, Canada et al. 1996, Sullivan et al. 2000, White et al. 1998, Eschenbach 1995 and Park 1997).

Ben-Horin and Kroll (2012) discussed two reasons for the preference of NPV ranking over the IRR ranking: "i. The NPV is an absolute measure of wealth, whereas IRR is a relative measure of wealth, and ii. The time value of money employed in calculating the NPV is the risk-adjusted cost of capital, which is a measure of the actual economic opportunity cost of the capital invested in the project. On the other hand, the time value of money employed in calculating the IRR is the IRR itself, which is an artifact of the project's cash flow and does not represent an economic alternative cost." These reasons are invalid for three reasons: a. NPV and IRR are estimates produced by the same DCF methods; b. they use the same NCF data and applying the time value of money; and c. both can be compared with the risk-adjusted cost of capital. When the IRR is above the risk-adjusted cost of capital the project is accepted.

One of the common practice is to estimate IRR from the NPVs by interpolation and at IRR the NPV becomes zero. From an analytical or mathematical perspective, NPV is a point estimate (at hurdle rate) whereas the IRR estimate involves evaluating a range or profile of NPVs at various discount rates to locate the discount rate that makes the NPV = 0. Instead of a point estimate (NPV), if a NPV profile review is undertaken, the adequacy or inadequacy of NPV could be better explained and appreciated. This paper investigates and presents numerical evidence to better appreciate the capital investment analysis criteria, NPV and IRR.

3. Methodology

Some of the key issues relating to CBA are discussed that would facilitate a better appreciation of the methodology.

Return 'of' and 'on' capital: Any investment decision involves estimation of two components of return: 1. return of capital (ROC) or recovery of capital invested; and 2. a desired return on invested capital (ROIC). The ROIC is measure of the return earned on capital invested. The DCF method makes use of the NCF data to estimate the ROC and ROIC.

Mathematically, IRR is the discount rate that makes the NPV = 0 and the BCR = 1. These relationships among IRR, NPV and BCR remain consistent for normal and non-normal NCF. Normal positive cash flow stream (normal or orthodox NCF) refers to non-negative net cash flow all through other than the investment year (Year 0). In the case of the non-normal NCF, the cash flow of a project changes sign more than once, e.g. if one or two negative flows are followed by some inflows. In these cases, the project may have more than one IRR and the NPV will be zero as many times as there are multiple IRR. Now, some of these controversies or criticisms relating the IRR and NPV as decision criteria are evaluated using some of the methodologies discussed below.

Estimation of IRR and NPV: Equation 1 is commonly being used to estimate the IRR and NPV.

$$NPV = \frac{CF1}{(1+r)^1} + \frac{CF2}{(1+r)^2} + ...\frac{CF_t}{(1+r)^t} - CF0 \qquad \text{Equation 1.}$$

Where, CF_1, CF_2… CF_t are the net cash inflow during periods 1 to t; CF_0 is the capital invested; 'r' is the discount rate. NPV is estimated with hurdle rate or cost of capital as the discount factor 'r'. IRR is identified by using a range of discount rates (r). The 'r' that makes the NPV = 0, is the IRR. While the IRR remains constant for a NCF, the NPV keeps changing for the same NCF with changes in the discount rate 'r' and the NPV is sensitive to discount rates.

In the Eq.1, the sum of the right-hand side (RHS) variables (other than the CF_0) is the discounted net cash inflow or the PV of net cash inflows (PVcf). PVcf minus CF_0 is the NPV as per Eq.2 that is derived by substituting PVcf on the RHS in Eq.1.

$$NPV = PV_{cf} - CF_0 \qquad \text{Equation 2}$$

This equation is useful to interpret the relationship between NPV and IRR. First, as per Eq.2, NPV is the unutilized PVcf after recovery of the capital (CF_0 or ROC) and recovery of ROIC at hurdle rate. The PVcf is also the remaining ROIC in absolute term after the recovery of ROC and a part of ROIC in percentage terms (hurdle rate). The PVcf keeps changing for every change in the discount rate (r). The discount rate used represents the ROIC. ROIC is the earning on the unrecovered balance of an investment. As such, the discount rate (ROIC) used in the denominators of each present value (PV) computation is critical in determining what the final NPV number will turn out to be. A small increase or decrease in the expected or desired ROIC (r) will have a considerable effect on the final output of NPV. Several interpretations can be made from the Eq.2 as below:

When r = IRR, the NPV will be '0' that indicates the full utilization of the PVcf to pay-off the CF_0 (ROC) and the highest possible ROIC (= IRR). With IRR as the ROIC, the investment income is optimized.

When r < IRR, the NPV is positive. The positive NPV represents the unutilized PVcf (see Eq.2). The lower discount rate (r less than IRR) is inadequate to fully utilize the PVcf and to maximize the ROIC (ROIC < IRR).

When r > IRR, the NPV is negative. Here, the PVcf is not adequate to support that higher ROIC (r higher than IRR) and therefore the negative NPV.

Both IRR and NPV are estimated by DCF method using the NCF data and are interrelated. The analysis is further expanded with the capital amortization analysis.

Capital amortization schedules (CAS) Methods: Capital amortization schedule is a table or chart showing how much of each periodic future income or return from an investment is going towards interest payments or return (ROIC) and capital (principal) recovery (ROC). CAS virtually accounts for the ROC

and ROIC before discounting. CAS analysis will indicate when the net cash inflow (NCF) is fully utilized (CB zero) to recover the ROC and a ROIC that remains invested. CB of >0 and <0 reveals that there is excess (unutilized) benefit at that ROIC or benefits not sufficient to support the required ROIC, respectively. The PV of the final CB is the NPV (see Arjunan 2017a) that will be 0 when the ROIC is equal to IRR or >0 when the ROIC < IRR (underestimate) or <0 when the required ROIC > IRR (unachievable). CAS estimation follows equation 3.

$$\sum_{t=1}^{n}(OB_t * (1+R) - NCF_t) = CB_t \qquad \text{Equation 3.}$$

Where OB is the opening balance of capital each year, 'r' is the interest rate (or return r), NCF is the net cash inflow or the intermediate net income per year, CB is the closing balance each year and 't' is year 1 to n (n is terminal or final year of project life or maturity of investment). The CB of a year is the OB of the next year. Arjunan (2017a) linked the CAS and DCF equations (eq. 2 and 3) and illustrated that the NPV is the PV of the final CB and the rate that makes the final CB zero is the IRR in CAS:

$$PVCB = \frac{CF_1}{(1+r)^1} - CF_0 = NPV \qquad \text{Equation 4}$$

Arjunan (2017c) also introduced a modified CAS (MCAS) method. MCAS method just eliminate the reinvestment income of NNCF that resolve the problem of multiple IRR and leads to unique IRR. That method is also used in the present analysis.

4. Results and discussions:

The discussions in this section is organized into four sub-headings: a. reinvestment of intermediate income; b. No IRR or negative IRR or multiple cases; c. mutually exclusive investment cases; d. return on total capital vs return on remaining capital; and e. concluding remarks on NPV vs IRR.

4.1 Reinvestment of intermediate income:

The most common assertion is that the IRR involves reinvestment of the intermediate income at IRR whereas the NPV at hurdle rate or cost of capital (CoC). The validity of this assertion or assumption is investigated first. For a given NCF or NNCF, the estimated IRR and NPV by DCF or CAS methods are the same but the estimates based on MCAS method is likely to be different for some of the NNCF cases. The CAS and MCAS methods are used as they are transparent and easy to interpret. An illustrative analysis of a NCF and a NNCF investment are presented in table 1. For investments A and B, IRR and NPV at 10% (assumed hurdle rate) are estimated both by CAS (DCF) and MCAS methods and furnished in table 1.

a. With normal NCF (investment A), the opening balance (OB) in all years (1 to 4) is negative (see table 1) and therefore there is only interest expenses and no interest income or reinvestment income is included. As there is no reinvestment, there is no difference between the IRRs (18.3%) and NPVs ($183.9) estimated under CAS and MCAS methods. There is neither reinvestment at IRR nor at hurdle rate (10%) in the case of NPV (see Table 1).

b. Under the NNCF (investment B), the OB is positive in year 3 that leads to reinvestment income (interest income or positive interest) included under the estimate. With inclusion of reinvestment income, the estimated IRR and NPV for the NNCF (B) are 26.9% and $10685, respectively. Contrarily, the estimated IRR and NPV for the same NNCF (B) under the MCAS method (that excludes reinvestment income) are 23.26% and $8850. This clearly illustrates that with some NNCF investment not only the IRR but also the NPV is an overestimate as they are equally affected by the reinvestment income. IRR and NPV estimated by MCAS method, that excludes reinvestment, are the appropriate estimates.

c. In summary: a. with normal NCF investments, there is no reinvestment of intermediate income, neither at IRR or at hurdle rate in the case of NPV; b. With NNCF investments, there is reinvestment with some (not all[3]) of the NNCFs (see Arjunan 2017c); c. the reinvestment is a problem associated with some NNCF[3] data and nothing to do with IRR or NPV, as assumed or asserted in most of the published

[3] CAS of a NNCF investment that has positive OB in one or more years, only will lead to reinvestment.

works; d. any modification in the estimation of IRR or NPV, based on reinvestment assumption without evidence is a fallacy; and e. MIRR, that assumes reinvestment, is a spurious estimate.

Table: 1. An Analysis of Reinvestment of Intermediate Income at IRR or Hurdle Rates

Year	0	1	2	3	4
A. NCF Investment - CAS at IRR: 18.03%					
NCF	-1000	300	400	500	300
Opening Capital		-1000	-880.3	-639.0	-254.2
Interest at IRR: 18.03%		-180	-159	-115	-46
Income		300	400	500	300
Closing Balance		-880.3	-639.0	-254.2	0.0
A. NCF Investment - CAS at 10%: PV of CB: $269.2 = NPV at 10%: $183.9					
Opening Capital		-1000.0	-800.0	-480.0	-28.0
Interest at Hurdle Rate: 10%		-100.0	-80.0	-48.0	-2.8
Income		300.0	400.0	500.0	300.0
Closing Balance		-800.0	-480.0	-28.0	269.2
A. NCF Investment - MCAS at IRR: 18.03%					
NCF	-1000	300	400	500	300
Opening Capital		-1000	-880.3	-639.0	-254.2
Interest at IRR: 18.03%		-180.3	-158.7	-115.2	-45.8
Income		300	400	500	300
Closing Balance		-880.3	-639.0	-254.2	0.0
A. NCF Investment - MCAS at 10%: PV of CB: $269.2 = NPV at 10%: $183.9					
Opening Capital		-1000.0	-800.0	-480.0	-28.0
Interest at Hurdle Rate: 10%		-100.0	-80.0	-48.0	-2.8
Income		300.0	400.0	500.0	300.0
Closing Balance		-800.0	-480.0	-28.0	269.2
B. NNCF Investment - CAS at IRR: 26.9%					
NNCF	-50000.0	50000.0	30000.0	-40000.0	30000.0
Opening Capital		-50000.0	-13474.8	12893.8	-23631.4
Interest at IRR: 26.9%		-13474.8	-3631.4	3474.8	-6368.6
Income		50000.0	30000.0	-40000.0	30000.0
Closing Balance		-13474.8	12893.8	-23631.4	0.0
B. NNCF Investment - CAS AT 10%: PV of the CB: $15645 = NPV at 10%: $10685					
Opening Capital		-50000.0	-5000.0	24500.0	-13050.0
Interest at Hurdle rate: 10%		-5000.0	-500.0	2450.0	-1305.0
Income		50000	30000	-40000	30000
Closing Balance		-5000.0	24500.0	-13050.0	15645.0
B. NNCF Investment - MCAS at IRR: 23.26%					
Opening Capital		-50000.0	-11632.0	15662.0	-24338.0
Interest at IRR: 23.26%		-11632.0	-2706.0	0.0	-5662.0
Income		50000.0	30000.0	-40000.0	30000.0
Closing Balance		-11632.0	15662.0	-24338.0	0.0
B. NNCF Investment - MCAS at 10%: PV of CB: $12950 = NPV at 10%: $8850.0					
Opening Capital		-50000.0	-5000.0	24500.0	-15500.0
Interest at Hurdle Rate: 10%		-5000.0	-500.0	0.0	-1550.0
Income		50000.0	30000.0	-40000.0	30000.0
Closing Balance		-5000.0	24500.0	-15500.0	12950.0

The problem of reinvestment and the resolution of multiple IRR are discussed next.

4.2 No IRR, Negative or Zero or Multiple IRR

Conceptually, if there are multiple IRRs, then the NPVs will also be zeros as many times as the number of IRRs. The problem of multiple IRR and corresponding multiple zero NPVs are caused by the NNCF data used under the DCF analysis. Similarly, the problem of 'no or negative or zero' IRRs is caused by the some of the NNCF data. The discussions in this section starts with the cases of "no or negative or zero IRR". The results of numerical analysis conducted are presented in table 2 and summarized here:

Table: 2. NCF and NNCFS - Estimated IRR and NPV by DCF/CAS and MCAS Methods

Details	NCF 1	NCF 2	NCF 3	NCF 4	NCF 5	NCF 6
Year 0	0	-1000	-1000	-10000	-10000	-50000
Year 1	1000	900	900	5000	5000	50000
Year 2	1500	-300	-300	-7000	-6000	30000
Year 3	2000	900	1000	7000	7000	-40000
Year 4	2500	-600	-600	4000	8000	30000
Sum of NCF (SNCF)	**7000**	**-100**	**0**	**-1000**	**4000**	**300**
IRR by CAS/DCF[4] Method	No IRR	No IRR	0.0%	-3.3%	11.1%	26.9%
NPV at 10% (CAS/DCF)	$5,359	-$163	-$88	-$3248	$310	$10,686
IRR by MCAS Method	No IRR	-7.4%	0.0%	-3.3%	11.1%	23.3%
NPV at 10% (MCAS)	$4781	-$186	-$118	-$3248	$310	$8845

- NCF 1 (see table 2) data does not have any capital investments and the NCF for investment year (zero year) is zero. As there is no capital investment, the NCF will not have a single negative flow (in year 0 or later). There is no need for the estimation of IRR or to recover capital invested (ROC) when there is no capital invested. Naturally, there will be no IRR (see NCF 1, Table 2). Such a NCF (with all positive cash flows) naturally leads to reinvestment of intermediate income as discussed in the previous section.
- With NCF 1 data (with all positive cash flows), the estimated NPV by CAS[5] or DCF method, that includes reinvestment income, is $5359. For the same NCF1 data, the estimated NPV by MCAS

[4] Estimates by CAS and DCF perfectly match (see Arjunan, 2017).
[5] Due to space constraints, the CAS and MCAS analyses are not furnished here but available with author.

method, that does not include reinvestment income, is $4781. The NPV is also misleading because the estimate includes reinvestment at the hurdle rate (CoC).

- The NCF 2 is a NNCF and the sum of NNCF is negative (-$100). With such NNCF data, there is no IRR under CAS or DCF method. A CAS analysis, that includes reinvestment income, reveals that at zero interest rate the CB is -$100, the NPV is -$100 at 0% and -$163 at 10%. A MCAS analysis, that excludes reinvestment income, indicates that the CB is zero at -7.4% (that is IRR) and NPV at 10% is -$186. With NNCF investments both IRR and NPV are distorted by the reinvestment income.
- The un-discounted NCF is 0 in the case of NCF 3 (a NNCF). The IRR is '0%' for NCF3 and the cash inflow completely off-sets the capital and operating costs before discounting. Such situations are quite normal and that does not mean IRR is an unsuitable criterion. There is difference between the NPVs at 10% under CAS/DCF (-$88) and MCAS methods (NPV is -$118) that is caused by the reinvestment income. The NPV cannot provide a better investment criterion in these cases.
- NCF 4 is again a NNCF and the sum of undiscounted NNCF is -$1000. The CAS or MCAS prepared at IRR of 0% leaves a CB of -$1000 under each method. However, at IRR of -3.3%, the CB is '0' but the ROIC is unrecovered and therefore the IRR is negative. There is no difference between NPVs at 10% estimated under both CAS / DCF and MCAS methods. Negative IRR is again a problem associated with NNCF.
- NCF 5 and 6 are also NNCFs with sum of their undiscounted NNCF positive. NCF 5 does not have reinvestment income as there is no positive OB under the CAS. The estimated IRR and NPV are consistent under CAS and MCAS method. However, with NCF 6, there is positive OB in year three that adds reinvestment income. The reinvestment income distorts both NPV ($10686 and $8845) and IRR (26.9% and 23.3%) under CAS/DCF and MCAS method.

These findings illustrate that the problem of "zero, no or negative IRR" are the problem associated with NNCF. The NNCF data affects NPV as well as IRR. The widely accepted "NPV rule" needs re-examination, given the fact that NPV does suffers with NNCF. MIRR is also not the appropriate solution

as that assumes reinvestment and leads to external rate of return (see Crean 2005, Kierulff, 2012, Cheremushkin, 2012).

Multiple IRR is mostly associated with NNCF particularly if there is reinvestment of intermediate income. Some NNCFs without reinvestment income will not lead to multiple IRR. Projects A (Damodaran, 2010) and B (Chen 2008) with NNCF data, available in public domain, are selected for this study. In both projects, the NNCF is negative at year '0' and again at year 4 in both projects. These data are used to estimate the NPVs and IRRs and the results are presented in Table 3 and summarized here:

The sum of undiscounted NCF (SNCF) for projects A and B are -$100 million and -$70 million, respectively (net loss or net income is negative even without discounting). No commercial investor will consider such investments. The following analysis is therefore an academic exercise to clarify some controversial issues:

a. First, the IRRs and NPVs estimated by CAS/DCF method are discussed (Table 3):

- NNCFs A and B lead to multiple IRR with multiple zero NPVs corresponding with the number of IRRs. For projects A and B, there are two IRRs of 6.6% and 36.55% (9.9% and 32.2%)[6]. The NPVs also are zero twice, once at 6.6% and again at 36.55% (9.9% and 32.2%).

- Assuming a hurdle rate of 10%, the NPV is $27.80 million ($0.4million). NPV at 20% is $52.47 million ($15.6 million) which is the highest NPV. The usual discounting using the hurdle rate (a point estimate) fails to capture such highest but spurious NPV dynamics and would not have even noticed the zero NPV at 6.6% (9.9When the NPV is zero at discount rates 6.6% (9.9%), it is abnormal to get a positive NPV of $52.47 million ($15.6 million) at 20%. Both NPV and IRR estimates are confusing and not helpful in decision to invest or not. It is not rational to argue that IRR suffers with multiple rates (two here) without acknowledging the data problem. NPV does suffer with multiplicity of NPVs (several variants) and inconsistent NPVs caused by the NNCF as does IRR.

[6] Figures in parenthesis correspond to project B.

Table: 3. NNCFs and Estimated IRRs and NPVs by CAS/DCF and MCAS Methods

Year	NCF A	Estimates	By CAS/DCF	By MCAS
0	-1000	IRR (1)	6.6%	-9.0%
1	800	IRR (2)	36.5%	NA
2	1000	NPV at 6.6%	0	-$142.2
3	1300	NPV at 36.5%	0	-$193.4
4	-2200	NPV at 10%	$27.8	-$157.1
Sum of NCF (SNCF)	-100	NPV at 20%	$52.5	-$183.3
		NPV at -9%	-$396.3	0
Year	NCF B	Estimates	By CAS/DCF	By MCAS
0	-580	IRR (1)	9.9%	-12.1%
1	530	IRR (2)	32.2%	NA
2	530	NPV at 9.9%	0	-$94.3
3	530	NPV at 32.2%	0	-$109.0
4	-1080	NPV at 10%	$0.4	-$94.8
Sum of NCF (SNCF)	-70	NPV at 20%	$15.6	-$105.7
		NPV at -12.1%	$-319.8	0

b. Second, the IRRs and NPVs estimated by MCAS method are discussed (Table 3):

MCAS method (see Arjunan, 2017b for details) eliminates reinvestment income and resolve the problem of multiple IRR and leads to a unique IRR. The results derived by the MCAS method is discussed here:

- With elimination of the controversial reinvestment income associated with NNCFs projects A and B, there are unique IRR of -9.0% and -12.1%, respectively. The negative IRR realistically reveals approximate capital loss of 9 or 12% (of the capital invested) and therefore the investment must be rejected. The NPVs at 10% are also negative in both projects A and B.
- With the current practice of DCF or CAS methods, had the "NPV rule" or ÏRR rule" adopted, both A and B would have been accepted as the NPVs at 10% are positive and at least one of the IRRs are above hurdle rate. The MCAS method clearly indicate capital loss that is the reality.

c. The modified IRR (MIRR) estimated for the two projects are 10.3% and 10.1%, respectively (not furnished in Table 3). The NPVs at MIRR as discount rate are $29.69 and $0.41. The MIRR, having assumed reinvestment, is higher than the IRR and the NPV at MIRR as discount rate is not zero and therefore MIRR is misleading. MIRR assumes boundless return as the investment rate is increasing

irrespective of the fact whether the NCF could internally support such a MIRR (see Arjunan 2017c). The MIRR is therefore higher than the IRR and the NPV at MIRR as discount rate is not zero and therefore MIRR is misleading. MIRR is not a solution because it is termed as an external rate of return by some authors (see Crean 2005, Kierulff, 2012, Cheremushkin, 2012).

In summary, both IRR and NPV suffers with multiple estimates with NNCF data but then the criticism is equally, rather more, applicable to NPV. As pointed earlier, the DCF or CAS analysis using unorthodox or abnormal or non-normal NCF, with negative or zero or negligible undiscounted value, affects the credibility or rationality of the analysis. The limitation of the input data (NNCF) and the DCF/CAS method must be clearly understood. The MCAS method, for NNCF as well as NCF, offers consistent and realistic estimates and resolves the problem of multiple IRR.

4.3 Mutually Exclusive Projects

Investment decision, to accept or reject or rank mutually exclusive projects, based on contradicting NPV or IRR criteria is another controversial area. The prevailing conventional wisdom is that the IRR is not suitable for evaluating mutually exclusive projects (for e.g. Osborne 2010, Kierulff 2012, World Bank 2013, Bora, 2015). The validity of this convention is critically evaluated in this section.

As discussed earlier, NPV is a point estimate (at hurdle rate) whereas the IRR involves evaluating a range or profile of NPVs at various discount rates to locate the discount rate that makes the NPV = 0. If a NPV profile review is undertaken, some of the inconsistent results of NPV and IRR could be explained (see Chen 2008). Profitability Index (PV per dollar of investment or BCR) and crossover rate (the rate at which the net present values of two projects are equal) can be considered as supplementary criteria in decision making. Table 4 presents the NCFs, estimated results of NPV, IRR, MIRR, Profitability Index (PI) and cross-over rate and the CAS for three set of two mutually exclusive projects each based on the data available in public domain (see sources below the table 4).

NPV profiles are prepared for all projects at 10% and at IRRs of both projects (the IRR of project A is used as discount rate for project B and vice versa) and the results furnished in Table 4 in three parts: 1. NCFs for three sets of mutually exclusive projects; 2. Estimated IRR, NPV, MIRR, Cross-over rate and Profitability Index; and 3. NPV Profiles of mutually exclusive projects selected.

The results are summarized and discussed here.

a. Among the three sets of mutually exclusive projects, in the first set the IRR and NPV consistently support project B. In the case of second set, IRR supports project 'B' but NPV supports 'A'. Similarly, under case 3, IRR supports project 'A' whereas NPV supports project 'B'. The conventional wisdom presumes that the NPV is the best criteria. Accordingly, project 'A' must be accepted under second set and project 'B' under the third set.
b. Now a review of the NPV profiles for each of the three sets reveals a different picture (Table 4). At IRR, the capital invested is fully recovered with a return equivalent to IRR. The NCF is fully utilized to pay-off ROC and ROIC. That's what the function of capital investment to maximize the ROIC and to recover the ROC.
c. Cross over rate, as the discount rate, makes the NPV same for both mutually exclusive projects. PI (equal to BCR) is another output from the DCF using the same NCF data. These two measurements could not solve the problems.

Based on the results presented Table 4, had the point estimate (at cost of capital) NPV is used, projects B, A and B under the first, second and third sets of projects, respectively, would have been accepted. However, these projects cannot be accepted without review of the NPV profile, created with a range of discount rates, and the ROC and ROIC (IRR) estimates. The selection of a project based on NPV alone is not rational. The results of ROC, ROIC and NPV profiles are extracted from Table 4 and summarized in Table 5 to better appreciate the appropriateness of ROC and ROIC based criteria for the mutually exclusive projects.

Table: 4. Estimated IRR, NPV, MIRR, cross-over rate and profitability index

Years / Criteria	Silber (2016)		Osborne (2010)		Damodaran (2010)	
	Project A	Project B	Project A	Project B	Project A	Project B
1. NCFs for the three sets of mutually exclusive projects						
0	-1000	-1000	-100	-100	-1000	-10000
1	0	320	30	50	350	3000
2	0	320	35	50	450	3500
3	300	320	45	40	600	4500
4	600	320	60	20	750	5500
5	900	320	-	-	-	-
2. Estimated IRR, NPV, MIRR, Cross-over rate and Profitability Index						
IRR	14.7%	18.0%	22.0%	25.4%	33.7%	20.9%
NPV $	194.03	213.05	30.99	30.49	653.13	2757.33
MIRR (10%, 10%)	14.0%	14.3%	17.7%	17.6%	24.7%	16.9%
Cross-over rate		8.8%		10.7%		19.3%
Profitability Index (BCR)	1.19	1.21	1.31	1.30	1.65	1.28
3. NPV Profiles of mutually exclusive projects selected						
Silber (2016)			Osborne (2010)		Damodaran (2010)	
Discount Rate	Project A: NPV	Project B: NPV	Discount Rate	Project A: NPV	Project B: NPV	Discount Rate	Project A: NPV	Project B: NPV
10.0%	194.03	213.05	10.0%	30.99	30.49	10.0%	653.13	2757.33
14.7%	**0.00**	**81.20**	20.0%	4.28	9.18	15.0%	467.94	1358.66
15.0%	-12.23	72.69	**22.0%**	**0.00**	**5.66**	20.0%	313.08	187.11
18.0%	**-115.50**	**0.00**	25.0%	-5.98	0.67	**20.9%**	**288.37**	**0.00**
20.0%	-175.35	-43.00	**25.4%**	**-6.78**	**0.00**	30.0%	71.20	-1647.35
			30.0%	-14.72	-6.74	**33.7%**	**0.00**	**-2188.64**

Table: 5. Summary of ROC, ROIC and NPV Profiles (Extracted from Table 3)

Projects		ROC at IRR	ROIC (= IRR)	NPV at 10%	NPV at Various rates		Accept / Reject
					ROIC	NPV $	
Project Set 1	A	1000	14.7%	194.03	18%	-115.5	Reject
	B	**1000**	**18.0%**	**213.05**	**18%**	**0**	**Accept**
Project Set 2	A	100	22.0%	30.99	25.4%	-6.78	Reject
	B	**100**	**25.4%**	**30.49**	**25.4%**	**0**	**Accept**
Project Set 3	**A**	**1000**	**33.7%**	**653.1**	**33.7%**	**0**	**Accept**
	B	10000	20.9%	2757.3	33.7%	-2188.6	Reject

a. CASs at interest rates > IRR, result in negative CBs in all cases. In these cases, the ROC is not fully recovered and the ROIC above IRR is not a feasible solution for these NCFs. NCFs are not adequate

to achieve ROICs higher than IRR. In all these cases, the NCFs remain as negative CBs. Again, the PV of the negative CB at that interest rate (> IRR) is the negative NPV (see Table 5).

b. NPV is zero at IRR and positive at 10% (hurdle rate) in all the cases studied as expected. NPV continues to remain positive for all discount rates below IRR and but shift to negative territory for discount rates above IRR. This indicates that, at IRR, the PV of benefits offsets the PV of capital invested. The positive NPV at hurdle rate indicates that the NCF stream has the potential to generate a higher rate of return equal to IRR, more than the hurdle rate. The NPV, therefore, reveals the cut off point for acceptance and not the maximum ROIC achievable with the NCF generated by the investment.

Under the revised criteria of ROC and ROIC and review of NPV profiles, project B, B and A are accepted contrary to the conventional wisdom. These projects have recovered the capital invested (ROC) and achieved the highest ROIC (IRR is the highest) among the mutually exclusive projects in each set. Projects A, A and B are rejected as their ROIC (IRR) are lower than their counterparts. Their NPVs are negative at the highest IRR achieved by their counterpart projects that are accepted. This finding rejects the conventional wisdom of acceptance of projects based on NPV (at cost of capital) criteria among the mutually exclusive projects. As discussed earlier, NPV, being the PV of the unutilized balance of the NCF, fails to provide a better information than IRR. It is concluded that the acceptance or rejection criterion for mutually exclusive projects is IRR and not NPV.

The analysis is further expanded and CASs are prepared for all the three sets of mutually exclusive projects with various interest rates applied on the capital balance. Capital amortization schedule shows as to how much of each periodic future income or return from an investment is going towards interest payments (ROIC) and capital recovery (ROC). CAS virtually accounts for the ROC and the ROIC. CAS analysis indicates whether the NCF is fully utilized as reflected by the CB of zero at a desired rate of return. For each project three sets of CASs are prepared using 10% rate (assumed cost of capital), and IRRs of project and A and B as rates for both projects A and B in each set. The results are presented in Table 6 and discussed below.

Table: 6. Capital Amortization Schedules – Mutually Exclusive Projects

Interest Rate	Year	Project A				Project B			
		Opening Balance	Interest accrued	Income	Closing[1] Balance	Opening Balance	Interest accrued	Income	Closing[1] Balance
a. Data from Silber NPV vs IRR: Projects A and B									
10.00%	1	-1000	-100	0	-1100	-1000	-100	320	-780
	2	-1100	-110	0	-1210	-780	-78	320	-538
	3	-1210	-121	300	-1031	-538	-54	320	-272
	4	-1031	-103	600	-534	-272	-27	320	21
	5	-534	-53	900	313	21	2	320	343
14.70% (Project A: IRR)	1	-1000	-147	0	-1147	-1000	-147	320	-827
	2	-1147	-168	0	-1315	-827	-122	320	-629
	3	-1315	-193	300	-1208	-629	-92	320	-401
	4	-1208	-177	600	-785	-401	-59	320	-140
	5	-785	-115	900	0	-140	-21	320	160
18.00% (Project B: IRR)	1	-1000	-180	0	-1180	-1000	-180	320	-860
	2	-1180	-213	0	-1393	-860	-155	320	-695
	3	-1393	-251	300	-1344	-695	-125	320	-500
	4	-1344	-242	600	-986	-500	-90	320	-270
	5	-986	-178	900	-264	-270	-49	320	0
b. Data from Osborne (2010): Projects A and B									
10.00%	1	-100	-10	30	80	-100	-10	50	-60
	2	80	-8	35	53	-60	-6	50	-16
	3	53	-5	45	13	-16	-2	40	22
	4	13	-1	60	**-45**	22	2	20	45
22.00% (Project A: IRR)	1	-100	-22	30	-92	-100	-22	50	-72
	2	-92	-20	35	-77	-72	-16	50	-38
	3	-77	-17	45	-49	-38	-8	40	-6
	4	-49	-11	60	0	-6	-1	20	12
25.40% (Project B: IRR)	1	-100	-25	30	-95	-100	-25	50	-75
	2	-95	-24	35	-84	-75	-19	50	-45
	3	-84	-22	45	-61	-45	-11	40	-16
	4	-61	-16	60	-17	-16	-4	20	0
c. Data from Damodaran (2010): Projects A and B									
10.00%	1	-1000	-100	350	-750	-10000	-1000	3000	-8000
	2	-750	-75	450	-375	-8000	-800	3500	-5300
	3	-375	-38	600	187	-5300	-530	4500	-1330
	4	187	19	750	956	-1330	-133	5500	4037
20.90% (Project B: IRR)	1	-1000	-209	350	-859	-10000	-2090	3000	-9090
	2	-859	-180	450	-589	-9090	-1900	3500	-7490
	3	-589	-123	600	-112	-7490	-1565	4500	-4555
	4	-112	-23	750	615	-4555	-952	5500	0
33.70% (Project A: IRR)	1	-1000	-337	350	-987	-10000	-3370	3000	-10370
	2	-987	-333	450	-870	-10370	-3495	3500	-10365
	3	-870	-293	600	-563	-10365	-3493	4500	-9358
	4	-563	-190	750	0	-9358	-3154	5500	-7011

Note: 1. Closing Balance '0' refers to full utilization of the NCF; negative refers to excess recovery; and positive refers to yet to be recovered. **(Figures are rounded off)**

a) First, with IRR as the rate, the NCF is fully utilized to pay off the ROC and ROIC and therefore the CB is '0'. The ROIC is the highest possible return (IRR). With 10% rate (hurdle rate), the CB for all projects reveals an unutilized NCF. The 10% rate fails to account for the full value of the NCFs to maximize the ROIC (equal to IRR) and ends up with unutilized NCF. Indirectly this means that, at hurdle rate, the estimated NPV just indicates whether the cut-off point (bottom line) is reached but fails to indicate the maximum ROIC that could be reached. Only IRR can reveal the full potential of the NCF streams that recover the ROC and achieve the maximum possible ROIC.

b) Second, the estimated IRRs of project A and B are used and the CASs are prepared for both projects in all the three sets of projects. The projects with lower IRR (may be with higher NPV at 10%) failed to recover the capital invested at higher IRR as that was achieved by the counterpart projects. For example, under the third set, project A has lower NPV at 10% than project B but IRR is higher than the project B. The CAS for project A reveals that the NCF is fully optimized to recover the ROC and a higher ROIC (IRR) and the CB is zero. Project B failed to recover the capital fully (negative CB) at the higher IRR achieved by project A. Optimization of investment income (NCFs) is better revealed by the IRR in the CAS.

This result further reinforces the earlier findings discussed above. It is therefore time for the investment analyst and executives to move away from the NPV as the best criterion to select among mutually exclusive project. IRR is the best criterion to select the best outcome from an investment and in capital budgeting decisions.

4.4 IRR as return on total capital vs return on declined balance of capital

The analysts can easily estimate IRR on the declining capital or on total capital, if they decide to do so. The IRR on the declining capital and on total capital invested for two hypothetical NCFs are estimated using CAS method and furnished in Table 7. As could be seen the IRRs for NCF 1 are 14.7% and 11.3% on declining capital (normal IRR) and the total capital invested, respectively. For NCF 2, the estimated IRRs are 24.9% and 20% on declining capital and total capital, respectively. With return on total capital, the interest income or return is constant for all years (see CAS B, interest row in Table 4). The investment

executives may be more convinced with return on total capital invested. The important point is that the declining capital is due to the recovery of part of the capital every year. The recovered capital can be invested elsewhere that would generate return (opportunity cost of capital). That income will compensate for the difference between return on declining balance and on the total capital.

Table: 7. IRR as Return on Declining Balance of Capital vs Total Invested Capital

NCF 1: CAS A	-100000	25000	30000	40000	50000.0
Opening Capital		-100000	-89697.6	-72881.1	-43592.9
Interest at 14.7%		-14697.6	-13183.4	-10711.8	-6407.1
Income		25000.0	30000.0	40000.0	50000.0
Closing Balance		-89697.6	-72881.1	-43592.9	0.0
IRR on Declining Capital					14.7%
NCF 1: CAS B	-100000	25000	30000	40000	50000.0
Opening Capital		-100000	-86250.0	-67500.0	-38750.0
Interest at 11.3%		-11250.0	-11250.0	-11250.0	-11250.0
Income		25000.0	30000.0	40000.0	50000.0
Closing Balance		-86250.0	-67500.0	-38750.0	0.0
IRR on Total Capital					11.3%
NCF 2: CAS A	-10000	3000	4000	5000	6000.0
Opening Capital		-10000	-8469.8	-5714.6	-1554.5
Interest at 24.9%		-1469.8	-1244.9	-839.9	-228.5
Income		3000.0	4000.0	5000.0	50000.0
Closing Balance		-8469.8	-5714.6	-1554.5	48217.0
IRR on Declining Capital					24.9%
NCF 2: CAS B	-10000	3000	4000	5000	6000
Opening Capital		-10000	-9000.0	-7000.0	-4000.0
Interest at 20%		-2000.0	-2000.0	-2000.0	-2000.0
Income		3000.0	4000.0	5000.0	6000.0
Closing Balance		-9000.0	-7000.0	-4000.0	0.0
IRR on Total Capital					20.0%

4.4 NPV versus IRR: As discussed in section 3, the same equation (Eq. 1) is used to estimate both IRR and NPV, using the same NCF data and the same DCF method. That being the case, the superiority or otherwise of both IRR and NPV depends on the quality of the input data (NCFs) and the perfection of the estimation method (DCF). Any superiority or deficiency of the IRR and NPV can be judged by the full information content and which one is most appropriate. The NCFs data, both normal and non-normal, relating to representative projects that are available in public domain (see source in Table 8) are used in this study. The estimated results are used to illustrate the myth and reality of the ongoing argument on the superiority or inferiority of IRR and NPV. The results are presented in Table 8 and interpreted below:

a. CAS analysis indicates that the sum of NCF is equal to the sum of ROC, ROIC and the closing capital (CC). When the NCF is fully utilized the CC becomes zero. The PV of CB is NPV (Arjunan 2017a) and therefore the NPV is zero.

b. CAS at IRR[7], as interest rate, leads to CBs of '0' in all cases studied. This result indicates that the NCF is fully utilized to pay of the ROC and highest possible ROIC at IRR. It can be also inferred that the NCF supports a maximum ROIC equal to IRR and the recovery of the ROC in full. The NPV is zero at IRR, as the interest rate, as expected.

c. CASs at interest rates < IRR, end up with positive CBs in all cases. In these cases, the ROC is fully recovered but the ROIC is lower than the IRR. NCF is not fully utilized to achieve the highest possible ROIC (i.e. IRR) and the unutilized NCFs remain as the CBs in all cases and the PV of the CB is the NPV.

d. An equation developed by Arjunan (2017a) reveals that the PV of the CB at an interest rate (< IRR) is the NPV at that rate (see Table 1) and the rate that makes the CB zero in the CAS is IRR. In capital budgeting or capital investment analysis the NPV does not indicate the value added or wealth created to the firm as argued by many authors or the economic return as strangely claimed by Tang and Tang (2003). The real value or wealth is NCF and not the NPV.

e. CASs at interest rates > IRR, result in negative CBs in all cases. In these cases, the ROC is not fully recovered and the ROIC above IRR is not a feasible solution for these NCFs. NCFs are not adequate to achieve ROICs higher than IRR. In all these cases, the NCFs remain as negative CBs. Again, the PV of the negative CB at that interest rate (> IRR) is the negative NPV (see Table 8).

f. At 10% discount rate (hurdle rate) the ROC is fully recovered (Table 5, for example, in the first case the capital investment $77,000). With ROIC at 10%, there is unutilized portion of the NCFs remaining as CB. Mathematically, with discount rates lower than IRR (or with lower interest rate), NPV fails to

[7] CAS for all other rates are estimated but full CAS at IRR only is furnished. According to Arjunan (2017a), IRR is the rate that makes the closing balance zero in the CAS.

indicate the maximum or exact ROIC possible for a NCF. NPV can at best reveals the ROIC in absolute terms ($).

Table: 8. Selected Normal and Non-normal NCFs and Estimated NPV, IRR and ROC

Year	CAS for various investments at IRR				NCF	Rates	Estimated values				
	Opening Balance (OB)	Interest charged (IC)	Cash inflow (NCF)	CB (CC = OB+IC-NCF)			CB (CC)	PV of CC = NPV	ROC[2]	ROIC (Sum of IC)	ROC + ROIC + CC = sum of NCF
1. Data Source: Rich, S.P. and Rose, J.T. (2014): $											
0		17.3%			-77000						
1	77000	13297	35000	55297	35000	10.0%	13363	10,040	77000	14637	105000
2	55297	9549	35000	29846	35000	15.0%	4430	2,913	77000	23570	105000
3	29846	5154	35000	0	35000	17.3%[1]	0	0	77000	28000	105000
Total		28000	105000	0		20.0%	-5656	-3,273	77000	33656	105000
2. Data Source: Osborne, M.J. (2010): Project B- $ million											
0		25.4%			-100						
1	100	25.4	50	75.4	50	10%	44.6	30.0	100	15.4	160
2	75.4	19.2	50	44.6	50	20%	19	9.0	100	41	160
3	44.6	11.3	40	15.9	40	25.4%[1]	0	0.0	100	60	160
4	15.9	4.1	20	0	20	30%	-19.26	-7.0	100	79.3	160
Total		60	160								
3. Data Source: Silber, W.L. (NPV vs IRR, site visited 2016): Project B: $											
0		18.0%			-1000						
1	1000	180.3	320	860.3	320	10.0%	343	213.0	1000	257	1600
2	860.3	155.1	320	695.4	320	15.0%	146	72.6	1000	454	1600
3	695.4	125.4	320	500.8	320	18.0%[1]	0	0.0	1000	600	1600
4	500.8	90.3	320	271.1	320	25.0%	-426	-139.6	1000	1026	1600
5	271.1	48.9	320	0	320						
Total		600	1600								
4. Non-normal NCF[2] – Data Source Damodaran (2010)											
0		36.5%			-1000						
1	1000	365.5	800	565.5	800	10.0%	40.7	28	1000	-140.7	900
2	566	206.7	1000	-227.8	1000	30.0%	-45.4	29	1000	-54.6	900
3	-228	-83.3	1300	-1611.1	1300	36.5%[1]	0	0	1000	-100	900
4	-1611	-588.9	-2200	0	-2200	40.0%	-66.4	-17	1000	-33.6	900
Total		-100	900								

1. IRR as the interest or discount rate (both have the same impact on NCF)
2. Multiple IRRs (36.5% and 6.5%) with similar impacts on CAS but the impact by one of the IRRs is presented here.

g. IRR explicitly indicates that the maximum return that is much above the hurdle rate. At IRR, the PV of benefit stream matches with the PV of capital cost (NPV = 0) and thereby reveals that the IRR fully captures the intrinsic value or internal return from the NCF. In the present case, the IRR is higher than the hurdle rate or ROIC of other potential investment available in the market.

h. As a point estimate (at a specific discount rate, hurdle rate), NPV fails to capture or reveal the highest rate of return that the NCF could support. NPV leaves it to the investor to guess what could be the maximum ROIC for that NCF stream. IRR iterates along the NPV profile at various discount rates and identify the NPV at which the ROC is fully recovered with the highest or maximum ROIC for that NCF. For example, when the NPV = 0 at IRR, that explicitly indicates that the NCF stream is sufficient to recover the capital invested (ROC) with a higher rate of return equivalent to the IRR. A smart investor will always look for a full recovery of the capital invested (ROC) with a highest rate of return for the NCF stream (see Damodaran, 2010) than restricting to the point estimate at 10% (hurdle rate) and then guessing the implicit higher return possible for that NCF

i. Finally, with the NNCF (case 5, Table 5), the total undiscounted income is less than the capital invested and investors will outright reject such project. The sum of ROC and ROIC is $900 when compared with the capital investment of $1000. There are multiple IRRs in this case but the results furnished relate to one of those IRRs (detail of multiple IRR is already discussed).

In summary, NPV at hurdle rate, being a static point estimate, provides incomplete information and does not indicate the full ROIC for the given NCF. At hurdle rate, NPV can be zero or positive or negative indicating that the NCF is fully utilized or not fully utilized or not adequate, respectively. IRR, in contrast, fully utilize the NCF (that makes NPV ‘0’) indicates the full ROIC plus ROC. Corporate management always strive hard to fully utilize the NCF and maximize the return or the profitability of the investment. IRR is the best indicator as that reveals the exact rate of return which is above the hurdle rate. This makes out a case, with stronger evidence, to continue with IRR as the primary criteria. These are the main reasons

for the private sector to prefer IRR and not NPV (see Osborne, 2010; Brealey et al. 2009; Bhattacharyya, 2004). NPV is a restricted estimate and provides incomplete information to support investment.

These findings make out a case to revise the Corporate Finance texts and CBA guidelines by the respective authorities and to recommend IRR as the best criterion for capital investment analysis. Most real-life investments will have a consistently orthodox or normal NCF. In those cases, the IRR and NPV will agree with each other in terms of a decision to accept or reject an investment. In the cases of abnormal NCF (case 4, Table 5), the total undiscounted income is less than the capital invested and investors will reject such project.

5. Summary and Conclusion:

In this paper, the controversial issues in CBA and capital investment analysis, viz. reinvestment of intermediate income, multiple IRR (including no or zero or negative IRR), the right criterion to rank mutually exclusive investments and the superiority of NPV vs IRR as a criterion, are evaluated and presented. This analysis makes use of data from different sources including some hypothetical cases. The approach primarily involves estimation of NPV and IRR by DCF/CAS and MCAS methods under different scenarios and discount rates. The estimated indicators are compared and investigated to identify solutions to some of those controversial issues. The main findings are summarized here below:

a. With normal NCF and some of the NNCF data, neither IRR nor the NPV involves reinvestment at rates equivalent to IRR or at hurdle rates, respectively. In those case, the present analytical evidence rejects the assumption of reinvestment at IRR or at hurdle rate (CoC).
b. Positive OB for one or more years under the CAS is possible with some NNCF (not all NCCF). Those positive OB triggers the reinvestment (interest income on positive OB) that leads to multiple IRR. The MCAS method eliminates the reinvestment income and thereby resolves the problem of multiple IRR and results in unique IRR.

c. Reinvestment of intermediate income and the associated multiple IRR are symptoms of some of the NNCF data and not associated with all NNCF and normal NCF. The NPV will be zero as many time as there are IRRs. The limitation of the input data (NCF) and the DCF method are the causes and the effects are inconsistent IRR and NPV. This must be clearly acknowledged rather than arguing as to which estimate (IRR and NPV) is appropriate.

d. The modified IRR (MIRR) is not a solution because it assumes boundless return as the investment rate is increasing irrespective of the fact whether the NCF could internally support such a MIRR. The PI, which is BCR, being another estimate from the DCF method cannot solve the problem.

e. In investment analysis, if the NCF does not have a single negative flow (in year 0 or 1), then there will not be a IRR because there is no ROC needed. Similarly, if the un-discounted NCF is 0, then there is no IRR which is quite normal. In such cases, no IRR does not mean IRR is an unsuitable criterion. NPV can be used to estimate the present value only for valuation and not as an investment criterion.

f. Between NPV and IRR, IRR is the only indicator that is estimated by fully utilizing the NCF stream. IRR explicitly indicates whether the NCF is fully utilized to recover the ROC and ROIC. NPV makes use of a part of NCF to cover the cost of capital. When NPV is positive or zero or negative that indicates that the NCF is not fully unutilized or fully utilized or not adequate to cover the cost of capital, respectively. With NNCF data both IRR and NPV suffers with multiple rates or values. DCF analysis using unorthodox or NNCF affects the consistency of both IRR and NPV:

g. **Mutually Exclusive Investments:**

- Among the mutually exclusive projects, NPV failed to identify the right and highest ROIC for the given NCFs whereas the IRR consistently reveals the projects with highest ROIC. When the project with higher NPV at hurdle rate is discounted by the rate, equivalent to the IRR achieved by the counterpart project, the NPV becomes negative. Whereas the higher IRR project's NPV remains zero. These findings evidently show that IRR is the appropriate criterion and reject the arguments that the NPV is the best criteria for mutually exclusive projects. Neither the cross over rate as

discount rate makes the NPV same for both mutually exclusive projects nor the PI (BCR) could provide a better solution.

- The present findings support the ROC and ROIC combined criterion to rank or select among the mutually exclusive projects and for independent projects. These findings reject the conventional wisdom of acceptance of projects based on NPV (at cost of capital) criteria for the mutually exclusive projects. This is an important recommendation that goes against the recommended approach in many finance and economic texts.

h. **Independent project:** The corporate management always targets to maximize the return or the profitability of the investment and consider the NPV at hurdle rate (10%) as the bottom line. IRR explores and estimates the potential maximum return even beyond the hurdle rate. IRR is, therefore, the best indicator as that reveals the exact rate of return which is above the hurdle rate. At IRR, the capital invested is fully recovered (ROC) with a return equivalent to IRR (ROIC). The NCF is fully utilized to pay-off the ROC and ROIC. The management objective is to maximize the ROIC and to recover the capital invested (ROC). The ROC and ROIC combined indicators reveals that IRR is the best criterion to continue with as the primary criteria in CBA, capital budgeting and investment analysis for independent and mutually exclusive projects.

In conclusion, these findings make out a case to consider revising the Corporate Finance texts and CBA guidelines by the respective authorities and to recommend IRR as the best criterion for both independent and mutually exclusive capital investment projects. NPV is a static point estimate and represent PV of the unutilized NCF. In the cases of abnormal NCF, if the total undiscounted income is less than the capital invested, investors must reject such project than conducting a CBA or investment analysis.

References

1. Arjunan, K. C., 2017a, A New Method to Estimate NPV from the Capital Amortization Schedule and an Insight into Why NPV Is Not the Appropriate Criterion for Capital Investment Decision, Available at SSRN: https://ssrn.com/abstract=2899648
2. Arjunan, K. C., 2017b, Modified IRR (MIRR) is a spurious criterion and should not be used in cost-benefit analysis (CBA) and investment analysis, http://ssrn.com/abstract=2942456
3. Berkovitch, E. and Israel, R., 2004, Why the NPV Criterion does not Maximize NPV, Review of Finance Studies (2004) 17 (1): 239-255.
4. Bhattacharyya, N, 2004, Why Do Managers Prefer IRR (January 15, 2004). Available at SSRN: https://ssrn.com/abstract=496482 or http://dx.doi.org/10.2139/ssrn.496482
5. Bora, B, Comparison between NPV and IRR, Int. Journal of Research in Finance & Marketing, vol.5, Issue 12 (Dec 2015), ISSN 2231-5985
6. Brealey, R.A. and S.C. Myers, 1996, Principles of Corporate Finance, McGraw–Hill, New York, 1996.
7. Brealey, R.A., Myers, S.C. and Allen, F., 2009, Principles of Corporate Finance, 9th Edition, McGraw-Hill.
8. Brigham, E. and Ehrhardt, M., 2004, Financial Management: Theory and Practice (11th Edition), South -Western College Publication.
9. Brigham, E.F. and J.F. Houston, 2016, Fundamentals of Financial Management, Concise Edition, 9th Edition, p.401
10. Brounen, D., De Jong, A., and Koedijk, K. (2004), Corporate Finance in Europe: Confronting Theory with practice, Financial management, 33(4), 71-101.
11. Canada, J.R., W.G. Sullivan, and J.A. White, 1996, Capital Investment Analysis for Engineering and Management, Prentice–Hall, Upper Saddle River, New Jersey, 1996.

12. Chen, J. H., 2008, Finding Multiple Internal Rates of Return for A Project with Non-Conventional Cash Flows: Utilizing Popular Financial/Graphing Calculators and Spreadsheet Software, College Teaching Methods & Styles Journal – September 2008 Volume 4, Number 9, p.31-42.
13. Cheremushkin, S.V. 2012. There is No Hidden Reinvestment Assumption in Discounting
14. Crean, M.J., 2005, Point of View: Revealing the True Meaning of the IRR via Profiling the IRR and Defining the ERR, Journal of Real Estate Portfolio Management, Vol. 11, No. 3, 2005
15. Cuthbert, J.R. and M. Cuthbert, 2012, Why IRR is an inadequate indicator of costs and returns in relation to PFI schemes, Critical Perspectives on Accounting, Volume 23, Issue 6, September 2012, Pages 420–433
16. Damodaran, A. 2010, Applied Corporate Finance, Wiley; 3 edition (March 8, 2010);
17. Damodaran, A. 2010, The Investment Principle, Investment Management and Financial Innovations, Volume 7, Issue 2, 2010, http://people.stern.nyu.edu/adamodar/pdfiles/cfovhds/invN.pdf
18. Dudley, C. L. 1972. "A Note on Reinvestment Assumptions in Choosing between Net Present Value and Internal Rate of Return," The Journal of Finance. Vol. 27. No. 4. pp. 907-915.
19. Eschenbach, T.G., 1995, Engineering Economy: Applying Theory to Practice, Irwin, Chicago, 1995. Formula and IRR: Logical and Mathematical Arguments, (January 10, 2012).
20. Hall, J. and S. Millard, 2010, Capital budgeting practices used by selected listed South African firms, SAJEMS NS 13 (2010) No 1 85.
21. Hazen, G.B. 2003, A New Perspective on Multiple Internal Rates of Return, The Engineering Economist, 2003, Volume 48, Number 1.
22. Juhász, L. 2011, Net Present Value Versus Internal Rate of Return, Economics & Sociology, Vol. 4, No 1, 2011, pp. 46-53.

23. Kelleher, J.C. and J.J. MacCormack. 2004. “Internal Rate of Return: A Cautionary Tale. Tempted by a Project with High Internal Rates of Return? Better Check Those Interim Cash Flows Again,” The McKinsey Quarterly – McKinsey & Co. August 2004. pp. 16-19.
24. Kierulff, H., 2012. “IRR: A Blind Guide”, American Journal of Business Education, July/August 2012 Volume 5, Number 4.
25. Moshe Ben-Horin & Yoram Kroll (2012): The Limited Relevance of the Multiple IRRs, The Engineering Economist, 57:2, 101-118
26. Osborne, M. 2010, A resolution to the NPV-IRR debate? Quarterly Review of Economics and Finance, 50 (2). pp. 234-239. ISSN 10629769.
27. Park, C. S. 2005, Internal Rate of Return (Multiple Rates of Return Problem), Lecture No. 20 http://users.tamuk.edu/kfgfa00/CEEN%203317/Lectures/Lecture%20No20.pdf, Fundamentals of Engineering Economics.
28. Park, C.S., 1997, Contemporary Engineering Economics, Addison–Wesley, Menlo Park, California, 1997.
29. Phalippou, L., 2009, The Hazards of Using IRR to Measure Performance: The Case of Private Equity, CFA Digest, May 2009, Volume 39 Issue 2.
30. Rich, S.P.; and J.T. Rose. (2014), ‘Re-examining an old question: Does the IRR method implicitly assume a reinvestment rate?’ Journal of Financial Education, 40(1/2), p.152-166.].
31. Silber, W.L., “NPV versus IRR”, http://people.stern.nyu.edu/wsilber/NPV%20Versus%20IRR.pdf
32. Sullivan, W.G., J.A. Bontadelli, E.M. Wicks, E.M. and E.P. Degarmo, 2000, Engineering Economy, Prentice–Hall, Upper Saddle River, New Jersey, 2000.
33. Tang, S. L. and Tang, H. J. 2003. The variable financial indicator IRR and the constant economic indicator NPV, The Engineering Economist, Vol. 48(No. 1): pp. 69.

34. The World Bank, 2013. Investment Project Financing, Economic Analysis, Guidance Note. OPSPQ, Washington, DC. http://siteresources.worldbank.org/PROJECTS/Resources/40940-1365611011935_/Guidance_Note_Economic_Analysis.pdf

35. Weber, T.A. 2014. On the (non-)equivalence of IRR and NPV, Journal of Mathematical Economics, Volume 52, May 2014, Pages 25–39.

36. White, D. and G. VanLandingham. 2015, Some of the common criticisms of IRR and NPV are reviewed here, Journal of Benefit-Cost Analysis Volume 6, Issue 2 July 2015, pp. 369-399.

37. White, J.A, K.E. Case, D.B. Pratt, and M.H. Agee, 1998, Principles of Engineering Economic Analysis, Wiley, New York, 1998.