

Chapter 9 COMPUTER ARITHMETIC

➤ ALU

Data are presented to the ALU in registers, and the results of an operation are stored in registers. These registers are temporary storage locations within the processor that are connected by signal paths to the ALU. The ALU may also set flags as the result of an operation. For example, an overflow flag is set to 1 if the result of a computation exceeds the length of the register into which it is to be stored. The flag values are also stored in registers within the processor. The control unit provides signals that control the operation of the ALU and the movement of the data into and out of the ALU.

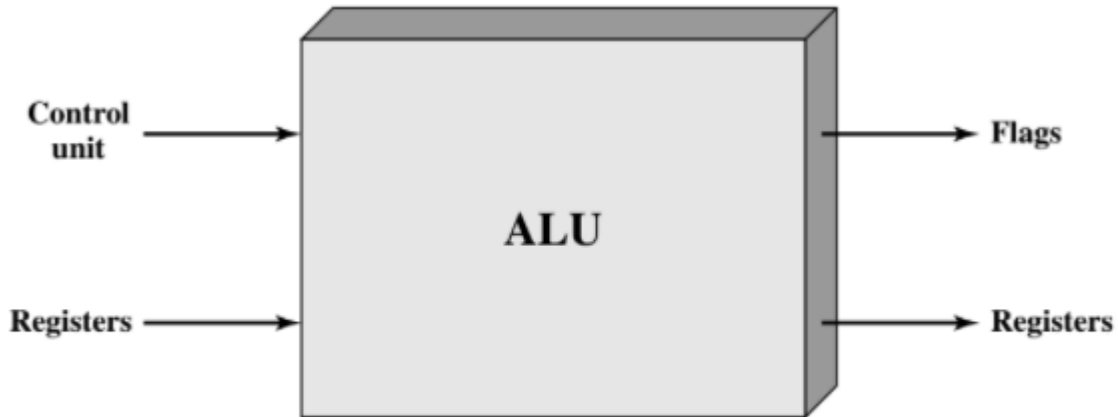


Figure 9.1 ALU Inputs and Outputs

➤ Addition of Numbers in 2's Complement Format

$\begin{array}{r} 1001 = -7 \\ +\underline{0101} = 5 \\ 1110 = -2 \\ \text{(a) } (-7) + (+5) \end{array}$	$\begin{array}{r} 1100 = -4 \\ +\underline{0100} = 4 \\ \underline{1}0000 = 0 \\ \text{(b) } (-4) + (+4) \end{array}$
$\begin{array}{r} 0011 = 3 \\ +\underline{0100} = 4 \\ 0111 = 7 \\ \text{(c) } (+3) + (+4) \end{array}$	$\begin{array}{r} 1100 = -4 \\ +\underline{1111} = -1 \\ \underline{1}1011 = -5 \\ \text{(d) } (-4) + (-1) \end{array}$
$\begin{array}{r} 0101 = 5 \\ +\underline{0100} = 4 \\ 1001 = \text{Overflow} \\ \text{(e) } (+5) + (+4) \end{array}$	$\begin{array}{r} 1001 = -7 \\ +\underline{1010} = -6 \\ \underline{1}0011 = \text{Overflow} \\ \text{(f) } (-7) + (-6) \end{array}$

Figure 9.3 Addition of Numbers in Twos Complement Representation

➤ Unsigned Binary Multiplication

The operation of the multiplier is as follows. Control logic reads the bits of the multiplier one at a time. If Q_0 is 1, then the multiplicand is added to the A register and the result is stored in the A register, with the C bit used for overflow. Then all of the bits of the C, A, and Q registers are shifted to the right one bit, so that the C bit goes into A_{n-1} , A_0 goes into Q_{n-1} and Q_0 is lost. If Q_0 is 0, then no addition is performed, just the shift. This process is repeated for each bit of the original multiplier. The resulting n -bit product is contained in the A and Q registers. A flowchart of the operation is shown in Figure 9.9.

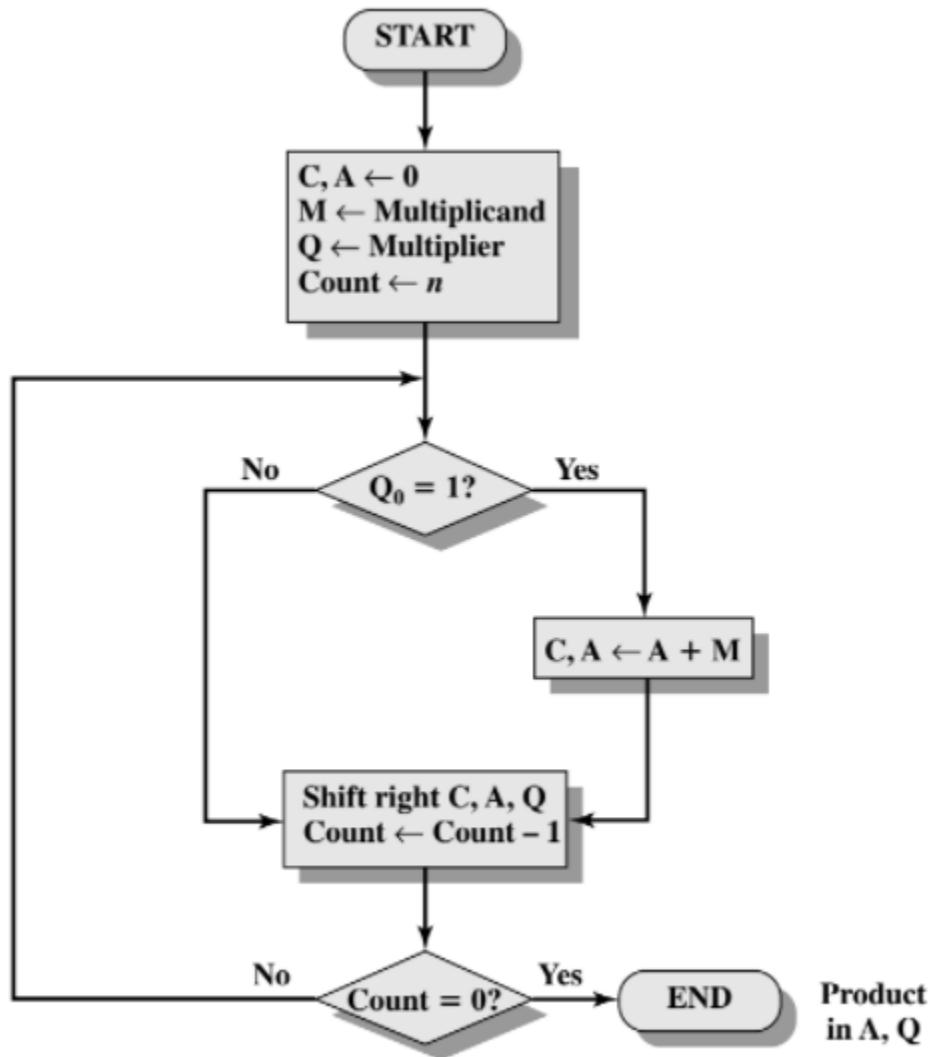


Figure 9.9 Flowchart for Unsigned Binary Multiplication

➤ **Booth Algorithm**

Booth's algorithm is depicted in Figure 9.12 and can be described as follows. As before, the multiplier and multiplicand are placed in the Q and M registers, respectively. There is also a 1-bit register placed logically to the right of the least significant bit of the Q register and designated its use is explained shortly.

The results of the multiplication will appear in the A and Q registers. A and Q_{-1} are initialized to 0. As before, control logic scans the bits of the multiplier one at a time. Now, as each bit is examined, the bit to its right is also examined. If the two bits are the same (1-1 or 0-0), then all of the bits of the A, Q, and Q_{-1} registers are shifted to the right 1 bit. If the two bits differ, then the multiplicand is added to or subtracted from the A register, depending on whether the two bits are 0-1 or 1-0. Following the addition or subtraction, the right shift occurs. In either case, the right shift is such that the leftmost bit of A, namely A_{n-1} not only is shifted into A_{n-2} but also remains in A_{n-1} , this is required to preserve the sign of the number in A and Q. It is known as an arithmetic shift, because it preserves the sign bit.

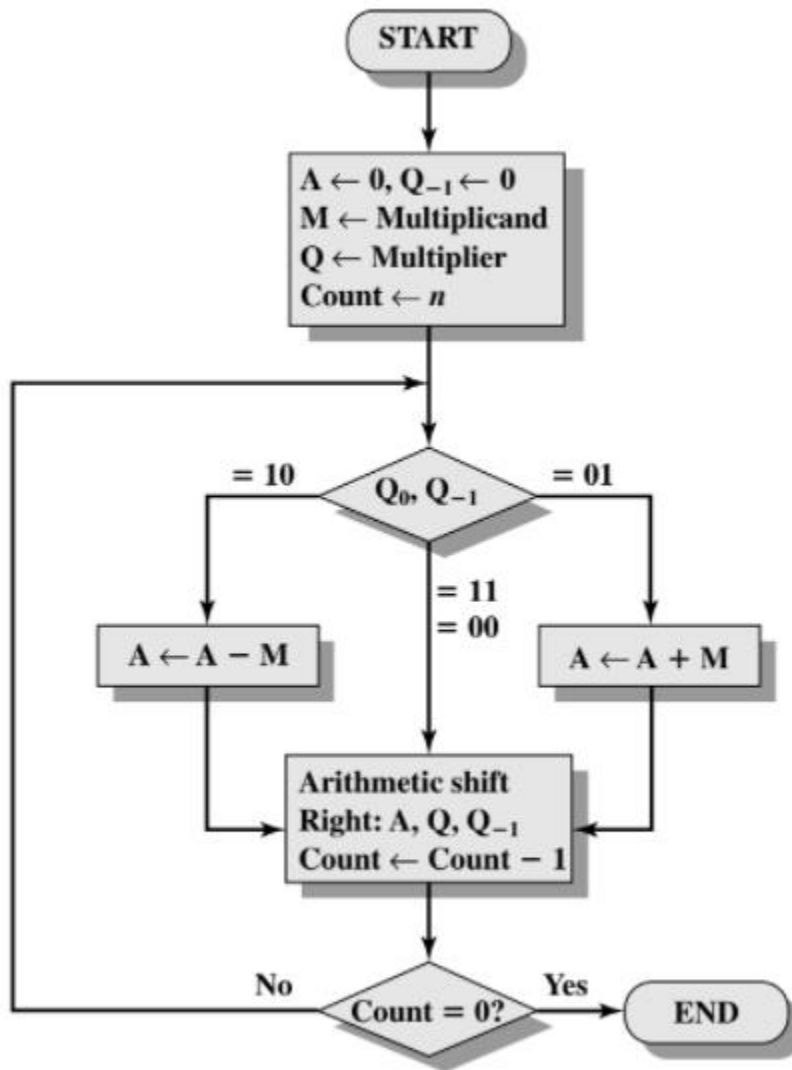


Figure 9.12 Booth's Algorithm for Twos Complement Multiplication

- **Example of Booth's Algorithm Multiplication : (7x3)**

A	Q	Q ₋₁	M		
0000	0011	0	0111	Initial values	
1001	0011	0	0111	A ← A - M Shift	} First cycle
1100	1001	1	0111		
1110	0100	1	0111	Shift	} Second cycle
0101	0100	1	0111	A ← A + M Shift	} Third cycle
0010	1010	0	0111		
0001	0101	0	0111	Shift	} Fourth cycle

Figure 9.13 Example of Booth's Algorithm (7×3)